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Coarse differentiation and quantitative nonembeddability for Carnot groups

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ABSTRACT

We give lower bound estimates for the macroscopic scale of coarse differentiability of Lipschitz maps from a Carnot group with the Carnot–Carathéodory metric (G, d_{cc}) to a few different classes of metric spaces. Using this result, we derive lower bound estimates for quantitative nonembeddability of Lipschitz embeddings of G into a metric space (X, d_X) if X is either an Alexandrov space with nonpositive or nonnegative curvature, a superreflexive Banach space, or another Carnot group that does not admit a biLipschitz homomorphic embedding of G. For the same targets, we can further give lower bound estimates for the biLipschitz distortion of every embedding $f : B(n) \to X$, where $B(n)$ is the ball of radius n of a finitely generated nonabelian torsion-free nilpotent group G. We also prove an analogue of Bourgain's discretization theorem for Carnot groups and show that Carnot groups have nontrivial Markov convexity. These give the first examples of metric spaces that have nontrivial Markov convexity but cannot biLipschitzly embed into Banach spaces of nontrivial Markov convexity.

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1. Introduction

Let G be a Carnot group endowed with a Carnot–Carathéodory metric and unit ball B_G and let (X, d_X) be some other metric space. Given a prescribed $\varepsilon \in (0,1)$, one can ask what is the largest $\rho(\varepsilon) > 0$ so that, given any Lipschitz function $f : B_G \to X$, there exists a subball $B(x,r) \subseteq B_G$ of radius $r \geqslant \rho(\varepsilon)$ and a map of canonical form $T : B_G \to X$ (whose form depends on the class of metric space to which X belongs) so that

$$\sup_{z \in B(x,r)} \frac{d_X(f(z), T(z))}{r} \leqslant \varepsilon \|f\|_{lip}. \tag{1}$$

Estimates of the form (1) originated from the work of [5] in the setting of normed linear spaces where T is an affine function. There, the authors named the property of having positive $\rho(\varepsilon)$ for any $\varepsilon \in (0,1)$ as the Uniform Approximation by Affine Property (or UAAP) and, they showed that $\mathrm{Lip}(X,Y)$, the space of Lipschitz functions from X to Y, has the UAAP if and only if one of the spaces $\{X,Y\}$ is finite dimensional and the other is superreflexive. For the case when Y is superreflexive, estimates of $\rho(\varepsilon)$ were given in [35] where they also used it to prove a restricted case of Bourgain's discretization theorem. For a similar statement concerning Lipschitz maps of finite dimensional vector spaces to general metric spaces see [1]. We generalize the results of [35] to the case when the domain is a Carnot group. These results belong in a class of methods that can be called quantitative or coarse differentiation. There is much research being done on this subject and its applications (cf. [15,20,19,34]).

We will provide some quantitative estimates for lower bounds of such ρ when (X,d) is a member of three classes: general metric spaces, superreflexive Banach spaces, and other Carnot groups. The canonical forms for the metric space classes are maps where horizontal lines are mapped to constant speed geodesics in the first case—this is not quite accurate, but will be made precise in Theorem 1.1—and group homomorphisms for the

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