Journal of Functional Analysis 266 (2014) 5126–5173

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

Equivalent conditions on periodic feedback stabilization for linear periodic evolution equations

Gengsheng Wang [a,1], Yashan Xu [b,*,2]

[a] School of Mathematics and Statistics, Wuhan University, Wuhan 430072, China
[b] School of Mathematical Sciences, Fudan University, KLMNS, Shanghai 200433, China

ARTICLE INFO

Article history:
Received 16 May 2013
Accepted 29 January 2014
Available online 26 February 2014
Communicated by C. De Lellis

Keywords:
Periodic evolution equations
Periodic feedback stabilization
Equivalent conditions
Attainable subspaces
Unique continuation properties
Poincaré map
Kato projection

ABSTRACT

This paper studies the periodic feedback stabilization for a class of linear T-periodic evolution equations. Several equivalent conditions on the linear periodic feedback stabilization are obtained. These conditions are related to the following subjects: the attainable subspace of the controlled evolution equation under consideration; the unstable subspace (of the evolution equation with the null control) provided by the Kato projection; the Poincaré map associated with the evolution equation with the null control; and two unique continuation properties for the dual equations on different time horizons $[0, T]$ and $[0, n_0T]$ (where n_0 is the sum of algebraic multiplicities of distinct unstable eigenvalues of the Poincaré map). It is also proved that a T-periodic controlled evolution equation is linear T-periodic feedback stabilizable if and only if it is linear T-periodic feedback stabilizable with respect to a finite-dimensional subspace. Some applications to heat equations with time-periodic potentials are presented.

* Corresponding author.
E-mail addresses: wanggs62@yeah.net (G. Wang), yashanxu@fudan.edu.cn (Y. Xu).

[1] The author was partially supported by the National Natural Science Foundation of China under grants 11161130003 and 11171264 and by the National Basis Research Program of China (973 Program) under grant 2011CB808002.
[2] The author was partially supported by the National Natural Science Foundation of China under grant 11371095.

http://dx.doi.org/10.1016/j.jfa.2014.01.029
0022-1236/

1. Introduction

1.1. The problem and the motivation

Consider the following controlled evolution equation:

$$y'(t) + Ay(t) + B(t)y(t) = D(t)u(t) \quad \text{in } \mathbb{R}^+ \triangleq [0,\infty). \tag{1.1}$$

Here and throughout this paper, we make the following assumptions.

($\mathcal{H}_1$) The operator $(-A)$, with its domain $\mathcal{D}(-A)$, generates a C_0 compact semigroup $\{S(t)\}_{t\geqslant 0}$ in a real Hilbert space H (identified with its dual) with its norm and inner product denoted by $\|\cdot\|$ and $\langle\cdot,\cdot\rangle$, respectively.

($\mathcal{H}_2$) The operator-valued function $B(\cdot) \in L^1_{loc}(\mathbb{R}^+; \mathcal{L}(H))$ is T-periodic, i.e., $B(t+T) = B(t)$ for a.e. $t \in \mathbb{R}^+$, where $T > 0$ and $\mathcal{L}(H)$ denotes the space of all linear bounded operators on H.

($\mathcal{H}_3$) The operator-valued function $D(\cdot) \in L^\infty(\mathbb{R}^+; \mathcal{L}(U,H))$ is T-periodic. Here U is also a real Hilbert space (identified with its dual) with its norm and inner product denoted by $\|\cdot\|_U$ and $\langle\cdot,\cdot\rangle_U$, respectively; and $\mathcal{L}(U,H)$ stands for the space of all linear bounded operators from U to H. Controls $u(\cdot)$ are taken from the space $L^2(\mathbb{R}^+; U)$.

For each $h \in H$, $s \geqslant 0$ and $u(\cdot) \in L^2(\mathbb{R}^+; U)$, Eq. (1.1) (over $[s,\infty)$) with the initial condition that $y(s) = h$ has a unique mild solution $y(\cdot; s, h, u) \in C([s,\infty); H)$. (See, for instance, Proposition 5.3 on page 66 in [13].) The following definitions about the periodic feedback stabilization will be used throughout this paper:

- Eq. (1.1) is said to be linear periodic feedback stabilizable (LPFS, for short) if there is a T-periodic $K(\cdot) \in L^\infty(\mathbb{R}^+; \mathcal{L}(H,U))$ such that the feedback equation

$$y'(t) + Ay(t) + B(t)y(t) = D(t)K(t)y(t) \quad \text{in } \mathbb{R}^+ \tag{1.2}$$

  is exponentially stable, i.e., there are two positive constants M and δ such that for each $h \in H$, the solution $y_K(\cdot; 0, h)$ to Eq. (1.2) with the initial condition that $y(0) = h$ satisfies that $\|y_K(t; 0, h)\| \leqslant Me^{-\delta t}\|h\|$ for all $t \geqslant 0$. Any such a $K(\cdot)$ is called an LPFS law for Eq. (1.1).
- Eq. (1.1) is said to be LPFS with respect to a subspace Z of U if there is a T-periodic $K(\cdot) \in L^\infty(\mathbb{R}^+; \mathcal{L}(H,Z))$ such that Eq. (1.2) is exponentially stable. Any such a $K(\cdot)$ is called an LPFS law for Eq. (1.1) with respect to Z.

Let

$$\mathcal{U}^{FS} \triangleq \{Z \mid Z \text{ is a subspace of } U \text{ s.t. Eq. (1.1) is LPFS w.r.t. } Z\}. \tag{1.3}$$