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On maximal subfactors from quantum groups

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ABSTRACT

Motivated by a conjecture about maximal intermediate subfactors, in this paper we show that subfactors from representations of universal quantum enveloping algebras with deformation parameter greater than zero are maximal. This gives continuous family of non-amenable subfactors which verify our conjecture. Using the same idea we also give a group theoretical description of our conjecture for subfactors related to representations of finite group.

1. Introduction

Let M be a factor represented on a Hilbert space and N a subfactor of M which is irreducible, i.e., $N' \cap M = \mathbb{C}$. Let K be an intermediate von Neumann subalgebra for the inclusion $N \subset M$. Note that $K' \cap K \subset N' \cap M = \mathbb{C}$, K is automatically a factor. Hence the set of all intermediate subfactors for $N \subset M$ forms a lattice under two natural operations $\wedge$ and $\vee$ defined by:

$$K_1 \wedge K_2 = K_1 \cap K_2, K_1 \vee K_2 = (K_1 \cup K_2)''.$$

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The commutant map $K \to K'$ maps an intermediate subfactor $N \subset K \subset M$ to $M' \subset K' \subset N'$. This map exchanges the two natural operations defined above.

Let $M \subset M_1$ be the Jones basic construction of $N \subset M$. Then $M \subset M_1$ is canonically isomorphic to $M' \subset N'$, and the lattice of intermediate subfactors for $N \subset M$ is related to the lattice of intermediate subfactors for $M \subset M_1$ by the commutant map defined as above.

Let G_1 be a group and G_2 be a subgroup of G_1. An interval sublattice $[G_1/G_2]$ is the lattice formed by all intermediate subgroups K, $G_2 \subseteq K \subseteq G_1$.

By cross product construction and Galois correspondence, every interval sublattice of finite groups can be realized as intermediate subfactor lattice of finite index. Hence the study of intermediate subfactor lattice of finite index is a natural generalization of the study of interval sublattice of finite groups. The study of questions which are related to intermediate subfactors has been very active in recent years (cf. [1,5,8,10,12], and [19] for a partial list).

In 1961 G.E. Wall conjectured that the number of maximal subgroups of a finite group G is less than $|G|$, the order of G (cf. [15]). In the same paper he proved his conjecture when G is solvable. In [11] it is proved that the number of maximal subgroups of a finite group G is less than $C|G|^{1.5}$ where C is a constant believed to be 1. In a recent breakthrough [6], counter examples to Wall's conjecture are found. However, Wall's conjecture is not that far off the mark in the sense that it is believed that the number of maximal subgroups of a finite group G is less than $|G|^{1+\alpha}$, where $\alpha > 0$ is small. In [20] we propose a subfactor generalization of Wall's conjecture. In view of recent breakthrough [6], it is necessary to modify this conjecture as follows:

Conjecture 1.1. *Let $N \subset M$ be an irreducible subfactor with finite index. Then the number of maximal intermediate subfactors is less than $(\dim(N' \cap M_1))^{1.5}$ where $N' \cap M_1$ is the second higher relative commutant of $N \subset M$.*

We note that since maximal intermediate subfactors in $N \subset M$ correspond to minimal intermediate subfactors in $M \subset M_1$, and the dimension of second higher relative commutant remains the same, the conjecture is equivalent to a similar conjecture as above with maximal replaced by minimal.

When $N \subset M$ comes from group G and its subgroup H, Conjecture 1.1 states that the number of maximal subgroups of G containing H is less than $n^{1.5}$, where n is the number of double cosets of H in G, and it is proved to be true for solvable group G in [7].

In [20,21] and [7], Conjecture 1.1 is verified for subfactors coming from certain conformal field theories and subfactors which are more closely related to groups and more generally Hopf algebras. The subfactors considered in these papers are finite depth. In [17] a class of irreducible subfactors are constructed from irreducible representations Λ of quantum universal enveloping algebra $U_q(g)$ with deformation parameter $q > 0$.