



ELSEVIER

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa



Products of generalized Nevanlinna functions with symmetric rational functions

S. Hassi*, H.L. Wietsma

Department of Mathematics and Statistics, University of Vaasa, P.O. Box 700, FI-65101 Vaasa, Finland

ARTICLE INFO

Article history:

Received 27 April 2011

Accepted 15 January 2014

Available online 3 February 2014

Communicated by J. Bourgain

Keywords:

Generalized Nevanlinna function

Hilbert space

Pontryagin space

Selfadjoint operator

Boundary triplet

Weyl function

Realization

ABSTRACT

New classes of generalized Nevanlinna functions, which under multiplication with an arbitrary fixed symmetric rational function remain generalized Nevanlinna functions, are introduced. Characterizations for these classes of functions are established by connecting the canonical factorizations of the product function and the original generalized Nevanlinna function in a constructive manner. Also, a detailed functional analytic treatment of these classes of functions is carried out by investigating the connection between the realizations of the product function and the original function. The operator theoretic treatment of these realizations is based on the notions of rigged spaces, boundary triplets, and associated Weyl functions.

© 2014 Elsevier Inc. All rights reserved.

1. Introduction

In the fifties M.G. Kreĭn introduced the classes of Stieltjes and inverse Stieltjes functions, denoted here by \mathcal{S} and \mathcal{S}^{-1} , as subclasses of the class of Nevanlinna functions. These functions were introduced in connection with investigations on the theory of the generalized resolvents and the theory of spectral functions of strings. M.G. Kreĭn showed that these classes have a simple characterization:

* Corresponding author.

E-mail addresses: sha@uwasa.fi (S. Hassi), rwietsma@uwasa.fi (H.L. Wietsma).

$$f(z) \in \mathcal{S} \iff f(z), zf(z) \in \mathcal{N},$$

with a similar characterization for the class of inverse Stieltjes functions due to the equivalence $f \in \mathcal{S}^{-1}$ if and only if $-f^{-1} \in \mathcal{S}$. Here \mathcal{N} denotes the class of (ordinary) Nevanlinna functions. In the seventies the class of Nevanlinna functions was generalized by M.G. Kreĭn and H. Langer to a class of generalized Nevanlinna functions with κ negative squares, denoted by \mathcal{N}_κ ; see [25,26]. One specific subclass of \mathcal{N}_κ , which was initially introduced for solving indefinite analogues of the Stieltjes moment problem, is the class \mathcal{N}_κ^+ defined as

$$f(z) \in \mathcal{N}_\kappa^+ \iff f(z) \in \mathcal{N}_\kappa, zf(z) \in \mathcal{N},$$

see [26]. This class extends the class of Stieltjes functions; it is known to be connected with the theory of spectral functions of a generalized string, see [29].

Further generalizations of the classes of Stieltjes (and inverse Stieltjes) functions are due to V.A. Derkach and M.M. Malamud:

$$f(z) \in \mathcal{S}_\beta^{+\kappa}(\alpha, \beta) \iff f(z) \in \mathcal{N}, \frac{z-\beta}{z-\alpha}f(z) \in \mathcal{N}_\kappa,$$

where $-\infty \leq \alpha < \beta < \infty$ (if $\alpha = -\infty$, then $z + \infty$ should be interpreted as being one). These classes were introduced to describe the selfadjoint extensions of a symmetric operator having gaps in its spectrum, see [9]. V.A. Derkach extended the Stieltjes (and inverse Stieltjes) classes also in a different direction:

$$f(z) \in \mathcal{N}_\kappa^{+\tilde{\kappa}} \iff f(z) \in \mathcal{N}_\kappa, zf(z) \in \mathcal{N}_{\tilde{\kappa}},$$

see [4]. All the preceding classes of complex functions can be seen as special cases of the classes $\mathcal{N}_\kappa^{\tilde{\kappa}}(r)$, which will be investigated in the present paper. The definition of the class $\mathcal{N}_\kappa^{\tilde{\kappa}}(r)$ involves an arbitrary symmetric rational function r and two indices $\kappa, \tilde{\kappa}$, which are used to describe the number of negative squares of the Nevanlinna kernels associated with the functions Q and rQ :

$$\mathcal{N}_\kappa^{\tilde{\kappa}}(r) = \{Q \in \mathcal{N}_\kappa: rQ \in \mathcal{N}_{\tilde{\kappa}}\}.$$

All the above mentioned special cases involve a simple symmetric rational function r of degree one; namely, either r or $1/r$ has the form $\frac{z-b}{z-a}$ or $z - b$, $a, b \in \mathbb{R}$.

The aim of this paper is to describe the characteristic properties of functions in the classes $\mathcal{N}_\kappa^{\tilde{\kappa}}(r)$ and to investigate relevant functional analytic connections they have in the area of operator and spectral theory. These classes of functions are applicable to several topics appearing in the literature for some elementary rational functions r . The results extend various previously known facts, which have been established for some special cases of the classes $\mathcal{N}_\kappa^{\tilde{\kappa}}(r)$.

The methods used in the present paper differ from those appearing in the above mentioned papers in some special cases; the approach is constructive and a basic tool

Download English Version:

<https://daneshyari.com/en/article/4590214>

Download Persian Version:

<https://daneshyari.com/article/4590214>

[Daneshyari.com](https://daneshyari.com)