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Stable sets and mean Li–Yorke chaos in positive entropy systems ☆

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ABSTRACT

It is shown that in a topological dynamical system with positive entropy, there is a measure-theoretically "rather big" set such that a multivariant version of mean Li–Yorke chaos happens on the closure of the stable or unstable set of any point from the set. It is also proved that the intersections of the sets of asymptotic tuples and mean Li–Yorke tuples with the set of topological entropy tuples are dense in the set of topological entropy tuples respectively.

1. Introduction

Throughout this paper, by a *topological dynamical system* (X,T) (t.d.s. for short) we mean a compact metric space X with a homeomorphism T from X onto itself. The metric on X is denoted by d. For a t.d.s. (X,T), the stable set of a point $x \in X$ is defined as

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$$W^s(x,T) = \left\{y \in X: \lim_{n\to+\infty} d\big(T^n x, T^n y\big) = 0\right\}$$

and the unstable set of x is defined as

$$W^u(x,T) = \left\{y \in X: \lim_{n\to+\infty} d\big(T^{-n} x, T^{-n} y\big) = 0\right\}.$$

Clearly, $W^s(x,T) = W^u(x,T^{-1})$ and $W^u(x,T) = W^s(x,T^{-1})$ for each $x \in X$. Stable and unstable sets play a big role in the study of smooth dynamical systems. Recently, there are many results related to the chaotic behavior and stable (unstable) sets in positive entropy systems (cf. [4,6,7,9,10,12,16]). So it indicates that stable and unstable sets are also important in the study of topological dynamical systems.

The chaotic behavior of a t.d.s. reflects the complexity of a t.d.s. Among the various definitions of chaos, Devaney's chaos, Li–Yorke chaos, positive entropy and the distributional chaos are the most popular ones. The implication among them has attracted a lot of attention. It is shown by Huang and Ye [17] that Devaney's chaos implies Li–Yorke one by proving that a non-periodic transitive t.d.s. with a periodic point is chaotic in the sense of Li and Yorke. In [4], Blanchard, Glasner, Kolyada and Maass proved that positive entropy also implies Li–Yorke chaos, that is if a t.d.s. (X,T) has positive entropy then there exists an uncountable subset S of X such that for any two distinct points $x, y \in S$,

$$\liminf_{n\to+\infty} d\big(T^n x, T^n y\big) = 0 \quad \text{and} \quad \limsup_{n\to+\infty} d\big(T^n x, T^n y\big) > 0.$$

We remark that the authors obtained this result using ergodic method, and for a combinatorial proof see [19]. Moreover, the result also holds for sofic group actions by Kerr and Li [20, Corollary 8.4].

In [6] Blanchard, Host and Ruette investigated the question if positive entropy implies the existence of non-diagonal asymptotic pairs. Among other things the authors showed that for positive entropy systems, many stable sets are not stable under T^{-1}. More precisely, if a T-invariant ergodic measure μ has positive entropy, then there exists $\eta > 0$ such that for μ-a.e. $x \in X$, one can find an uncountable subset F_x of $W^s(x,T)$ satisfying that for any $y \in F_x$,

$$\liminf_{n\to+\infty} d\big(T^{-n} x, T^{-n} y\big) = 0 \quad \text{and} \quad \limsup_{n\to+\infty} d\big(T^{-n} x, T^{-n} y\big) \geqslant \eta.$$

Distributional chaos was introduced in [28], and there are at least three versions of distributional chaos in the literature (DC1, DC2 and DC3), see [2] for the details. It is known that positive entropy does not imply DC1 chaos [27]. In [29] Smítal conjectured that positive entropy implies DC2 chaos, and Oprocha showed that this conjecture holds for minimal uniformly positive entropy systems [23]. Very recently, Downarowicz [9] proved that positive entropy indeed implies DC2 chaos. Precisely, if a t.d.s. (X,T) has