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Multiple solutions for semi-linear corner degenerate elliptic equations ☆

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ABSTRACT

The present paper is concerned with the existence of multiple solutions for semi-linear corner-degenerate elliptic equations with subcritical conditions. First, we introduce the corner type weighted p-Sobolev spaces and discuss the properties of continuous embedding, compactness and spectrum. Then, we prove the corner type Sobolev inequality and Poincaré inequality, which are important in the proof of the main result.

1. Introduction

Write $\mathbb{M} = [0,1) \times X \times [0,1)$ as a local model of stretched corner-manifolds (i.e. manifolds with corner singularities) with dimension $N = n + 2 \geqslant 3$. Here X is a closed compact sub-manifold of dimension n embedded in the unit sphere of $\mathbb{R}^{n+1}$. Let $\mathbb{M}_0$ denote the interior of $\mathbb{M}$ and $\partial\mathbb{M} = \{0\} \times X \times \{0\}$ denote the boundary of $\mathbb{M}$. The so-called corner-Laplacian is defined as

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$$\Delta_{\mathbb{M}} = (r\partial_r)^2 + (\partial_{x_1})^2 + \cdots + (\partial_{x_n})^2 + (rt\partial_t)^2,$$

which is a degenerate elliptic operator on the boundary $\partial\mathbb{M}$. The present paper is concerned with the existence of multiple weak solutions for the following Dirichlet problem

$$\begin{cases} -\Delta_{\mathbb{M}} u = g(z,u), & z := (r,x,t) \in \mathbb{M}_0, \\ u = 0 \quad \text{on } \partial\mathbb{M}. \end{cases} \tag{1.1}$$

Our main result can be stated as follows.

Theorem 1.1. *Let $g(z,u) : \mathbb{M} \times \mathbb{R} \to \mathbb{R}$ be a Carathéodory function with the following assumptions*:

(H-1) *Let $g(z,u)$ be odd, i.e. $g(z,-u) = -g(z,u)$;*
(H-2) $2 < p < 2^* = \frac{2N}{N-2}$ *and there exists a constant $C_0 > 0$ such that the following estimate holds almost everywhere*

$$\left|g(z,u)\right| \leqslant C_0\left(1 + |u|^{p-1}\right);$$

(H-3) *For the primitive $G(\cdot,u) = \int_0^u g(\cdot,v)\,dv$, there exist $q > 2$ and a constant R_0 such that for almost every $z \in \mathbb{M}$ and $|u| \geqslant R_0$ we have*

$$0 < qG(z,u) \leqslant g(z,u)u.$$

Then the Dirichlet problem (1.1) admits infinity many weak solutions in the corner type weighted Sobolev space $\mathcal{H}_{2,0}^{1,(\frac{N-1}{2},\frac{N}{2})}(\mathbb{M})$.

To show this result, methods of variational theory are employed, which can be traced back to Ambrosetti and Rabinowitz [1] in 1973, and Rabinowitz [11] in 1974. In [2], Bartolo, Benci and Fortunato proved optimal multiplicity results in the case of degenerate critical values. All these results also can be found in book [15]. Authors studied the Dirichlet problem of semi-linear elliptic equations on stretched cone in [3] and [4]. The corresponding cone Laplacian $\Delta_{\mathbb{B}} = (x_1\partial_{x_1})^2 + (\partial_{x_2})^2 + \cdots + (\partial_{x_n})^2$, which is degenerate at $x_1 = 0$. This kind of operator is a simple example of conical differential operators. Also the authors studied similar nonlinear problem in [5] for the edge Laplacian $\Delta_{\mathbb{E}} = (x_1\partial_{x_1})^2 + (\partial_{x_2})^2 + \cdots + (\partial_{x_n})^2 + (x_1\partial_{y_1})^2 + \cdots + (x_1\partial_{y_q})^2$ with edge singularity at $x_1 = 0$. On the other hand, the pseudo-differential operators with conical singularities and edge singularities have been wildly studied from various motivations by Egorov and Schulze [6], Schulze [13], Schrohe and Seiler [12], Melrose and Mendoza [9] and Mazzeo [8]. In this paper, we pursue further study for the existence of solutions to semi-linear degenerate elliptic equations on manifold with corner singularities. Here the so-called corner Laplacian $\Delta_{\mathbb{M}} = (r\partial_r)^2 + (\partial_{x_1})^2 + \cdots + (\partial_{x_n})^2 + (rt\partial_t)^2$ is degenerate at both $r = 0$ and $t = 0$, which is named after the local structure of manifold with corner