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On the essential spectrum of the Laplacian and the drifted Laplacian

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ABSTRACT

This paper concerns the L^2 essential spectrum of the Laplacian Δ and the drifted Laplacian Δ_f on complete Riemannian manifolds endowed with a weighted measure $e^{-f}\,d\,\mathrm{vol}_g$. We prove that the essential spectrum of the drifted Laplacian Δ_f is $[0,\infty)$ provided the Bakry–Émery curvature tensor Ric_f is nonnegative and f has sublinear growth. When $\mathrm{Ric}_f \geqslant \frac{1}{2}g$ and $|\nabla f|^2 \leqslant f$, we show that the essential spectrum of the Laplacian is also $[0,\infty)$. During the proofs of these results, the f-volume growth estimate plays an important role and may be of independent interest.

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1. Introduction

The spectrum of the Laplacian of complete non-compact manifolds has been studied and computed for a large class of manifolds in the past two decades. When the manifold has a soul whose exponential map is a diffeomorphism, supposing nonnegative sectional curvature and some other additional conditions, Escobar [5] and Escobar and Freire [6] proved that the L^2 spectrum of the Laplacian is $[0,\infty)$. The additional conditions assumed by them were proved to be unnecessary by Zhou [15].

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If the manifold has a pole, the essential spectrum of the Laplacian was proved to be $[0,\infty)$ by Li [8] (supposing the Ricci curvature nonnegative) and by Chen and Lu [3] (supposing the radial sectional curvature nonnegative). The same result was proved by Donnelly [4] when the Ricci curvature is nonnegative and the volume has Euclidean growth.

Without having to assume the existence of a pole, Wang [13] proved that the essential spectrum L^p of the Laplacian is $[0,\infty)$, for $p \in [1,\infty)$, provided the Ricci curvature is bounded below by $-\delta/r^2$, where r is the distance from a fixed point on the manifold and $\delta > 0$ depends only on the dimension. This result was generalized by Lu and Zhou [9], assuming only that $\underline{\lim}_{r\to\infty} \mathrm{Ric} = 0$. Both works proved initially that the L^1 essential spectrum is $[0,\infty)$ and extended this fact to L^p, $p \in [1,\infty]$, by using a result proved by Sturm [12] which asserted the p-independence of the spectrum when the volume has uniform sub-exponential growth and the Ricci curvature is bounded below.

Circumventing the use of Sturm's Theorem [12] by a generalization of Classical Weyl's Criterion, Charalambous and Lu [1] extended the results previously obtained in L^2 to the case where Ric is asymptotically nonnegative and $\overline{\lim}_{r\to\infty} \Delta r \leqslant 0$.

In many occasions, it is natural to consider a manifold M endowed with a weighted measure of the form $e^{-f} d\,\mathrm{vol}_g$, where f is a smooth function. We have the drifted Laplacian Δ_f, defined by

$$\Delta_f u = \Delta u - \langle \nabla f, \nabla u \rangle,$$

associated with the differential of the volume form of $e^{-f} d\,\mathrm{vol}_g$ the same way Δ is associated with $d\,\mathrm{vol}_g$. Moreover, Δ_f is a self-adjoint operator on the space L^2_f of square integrable functions on M with respect to the measure $e^{-f} d\,\mathrm{vol}_g$. A natural extension of the Ricci curvature tensor to this new context is the Bakry–Émery Ricci curvature tensor, given by

$$\mathrm{Ric}_f = \mathrm{Ric} + \mathrm{Hess}\, f.$$

Manifolds with the Bakry–Émery Ricci curvature bounded below have been studied by many authors in recent years, with the special objective of obtaining similar results to the case where the Ricci curvature is bounded below. Many results have been obtained, particularly some interesting volume estimates and splitting theorems, which can be found in [14] and [11]. In this paper, we apply some of these results to the analysis of the drifted Laplacian Δ_f. A natural context for the Bakry–Émery Ricci curvature tensor is the study of *gradient Ricci solitons*, manifolds where $\mathrm{Ric}_f = \lambda g$, for some constant λ and function f.

Our main goal in this paper is to determine the L^2 and L^2_f essential spectra of $-\Delta$ and $-\Delta_f$ respectively, supposing some lower bounds for Ric_f and imposing conditions on f.

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