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Hardy inequality and asymptotic eigenvalue distribution for discrete Laplacians

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Abstract

In this paper we study in detail some spectral properties of the magnetic discrete Laplacian. We identify its form-domain, characterize the absence of essential spectrum and provide the asymptotic eigenvalue distribution.
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Keywords: Magnetic discrete Laplacian; Locally finite graphs; Self-adjointness; Unboundedness; Semi-boundedness; Spectrum; Spectral graph theory; Asymptotic of eigenvalues; Essential spectrum

Contents

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1. Introduction

The uncertainty principle is a central point in quantum physics. It can be expressed by the following Hardy inequality:

$$\left(\frac{n-2}{2}\right)^2 \int_{\mathbb{R}^n} \left|\frac{1}{|x|} f(x)\right|^2 dx \leqslant \int_{\mathbb{R}^n} |\nabla f|^2 \, dx = \langle f, -\Delta_{\mathbb{R}^n} f\rangle, \quad \text{where } n \geqslant 3, \tag{1.1}$$

and $f \in \mathcal{C}_c^\infty(\mathbb{R}^n)$. Roughly speaking, the Laplacian controls some local singularities of a potential. In this paper, we investigate which potentials a discrete Laplacian is able to control. Obviously, since the value of a potential on a vertex has to be finite, we will not focus on local singularities. However, unlike in the continuous case, we will control potentials that explode at infinity.

We start with some definitions and fix our notation for graphs. We refer to [6,5,35] for surveys on the matter. Let $\mathscr{V}$ be a countable set. Let $\mathscr{E} := \mathscr{V} \times \mathscr{V} \to [0, \infty)$ and assume that

$$\mathscr{E}(x, y) = \mathscr{E}(y, x), \quad \text{for all } x, y \in \mathscr{V}.$$

We say that $G := (\mathscr{E}, \mathscr{V})$ is an unoriented weighted graph with *vertices* $\mathscr{V}$ and *weighted edges* $\mathscr{E}$. In the setting of electrical networks, the weights correspond to the conductances. We say that $x, y \in \mathscr{V}$ are *neighbors* if $\mathscr{E}(x, y) \neq 0$ and denote it by $x \sim y$. We say that there is a *loop* in $x \in \mathscr{V}$ if $\mathscr{E}(x, x) \neq 0$. The set of *neighbors* of $x \in \mathscr{E}$ is denoted by

$$\mathscr{N}_G(x) := \{y \in \mathscr{E}, \ x \sim y\}.$$

The *degree* of $x \in V$ is by definition $|\mathscr{N}_G(x)|$, the number of neighbors of x. A graph is *locally finite* if $|\mathscr{N}_G(x)|$ is finite for all $x \in V$. We also need a weight on the vertices

$$m : \mathscr{V} \to (0, \infty).$$

Finally, as we are dealing with magnetic fields, we fix a phase

$$\theta : \mathscr{V} \times \mathscr{V} \to [-\pi, \pi], \quad \text{such that} \quad \theta(x, y) = -\theta(y, x).$$

We set $\theta_{x,y} := \theta(x, y)$. A graph is *connected*, if for all $x, y \in V$, there exists an *x–y-path*, i.e., there is a finite sequence

$$(x_1, \ldots, x_{N+1}) \in \mathscr{V}^{N+1} \quad \text{such that} \quad x_1 = x, \qquad x_{N+1} = y \quad \text{and} \quad x_n \sim x_{n+1},$$

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