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Integral operators on the Oshima compactification of a Riemannian symmetric space ☆

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ABSTRACT

Consider a Riemannian symmetric space $\mathbb{X} = G/K$ of non-compact type, where G is a connected, real, semisimple Lie group, and K a maximal compact subgroup. Let $\widetilde{\mathbb{X}}$ be its Oshima compactification, and $(\pi, \mathrm{C}(\widetilde{\mathbb{X}}))$ the left-regular representation of G on $\widetilde{\mathbb{X}}$. In this paper, we examine the convolution operators $\pi(f)$ for rapidly decaying functions f on G, and characterize them within the framework of totally characteristic pseudodifferential operators, describing the singular nature of their kernels. As a consequence, we obtain asymptotics for heat and resolvent kernels associated to strongly elliptic operators on $\widetilde{\mathbb{X}}$. As a further application, a regularized trace for the operators $\pi(f)$ can be defined, yielding a distribution on G which can be interpreted as a global character of π, and is given by a fixed point formula analogous to the Atiyah–Bott character formula for an induced representation of G.

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1. Introduction

Let $\mathbb{X}$ be a Riemannian symmetric space of non-compact type. Then $\mathbb{X}$ is isomorphic to G/K, where G is a connected, real, semisimple Lie group, and K a maximal compact subgroup. Consider further the Oshima compactification $\widetilde{\mathbb{X}}$ of $\mathbb{X}$ [16], which is a simply connected, closed, real-analytic manifold carrying an analytic G-action. The orbital decomposition of $\widetilde{\mathbb{X}}$ is of normal crossing type, and the open orbits are isomorphic to G/K, the number of them being equal to 2^l, where l denotes the rank of G/K. In this paper, we study integral operators of the form

$$\pi(f) = \int_G f(g)\pi(g)d_G(g), \tag{1}$$

where π is the regular representation of G on the Banach space $\mathrm{C}(\widetilde{\mathbb{X}})$ of continuous functions on $\widetilde{\mathbb{X}}$, f a smooth, rapidly decreasing function on G, and d_G a Haar measure on G. These operators play an important role in representation theory, and our interest will be directed towards the elucidation of the microlocal structure of the operators $\pi(f)$. Since the underlying group action on $\widetilde{\mathbb{X}}$ is not transitive, the operators $\pi(f)$ are not smooth, and the orbit structure of $\widetilde{\mathbb{X}}$ is reflected in the singular behavior of their Schwartz kernels. As it turns out, the operators in question can be characterized as totally characteristic pseudodifferential operators, a class which was first introduced in [15] in connection with boundary problems. In fact, if $\widetilde{\mathbb{X}}_\Delta$ denotes a component in $\widetilde{\mathbb{X}}$ isomorphic to G/K, we prove that the restrictions

$$\pi(f)_{|\overline{\widetilde{\mathbb{X}}_\Delta}} : \mathrm{C}^\infty_{\mathrm{c}}(\overline{\widetilde{\mathbb{X}}_\Delta}) \to \mathrm{C}^\infty(\overline{\widetilde{\mathbb{X}}_\Delta})$$

of the operators $\pi(f)$ to the manifold with corners $\overline{\widetilde{\mathbb{X}}_\Delta}$ are totally characteristic pseudodifferential operators of class $\mathrm{L}^{-\infty}_b$. A similar structure theorem was already obtained in [18] for integral operators on prehomogeneous vector spaces, but only away from the set of singular points of the complement of the open orbit. In the present case, we are able to achieve a complete description of the operators $\pi(f)$ on $\overline{\widetilde{\mathbb{X}}_\Delta}$ even near the corners due to the fact that the orbital decomposition of $\widetilde{\mathbb{X}}$ is of normal crossing type.

As a first application, we employ the structure theorem to examine the holomorphic semigroup generated by a strongly elliptic operator Ω associated to the regular representation $(\pi, \mathrm{C}(\widetilde{\mathbb{X}}))$ of G, as well as its resolvent. Since both the holomorphic semigroup and the resolvent can be characterized as operators of the form (1), they can be studied applying our structure theorem, and relying on the theory of elliptic operators on Lie groups [19] we obtain a description of the asymptotic behavior of the semigroup and resolvent kernels on $\widetilde{\mathbb{X}}_\Delta \simeq \mathbb{X}$ at infinity. In the particular case of the Laplace–Beltrami operator on $\mathbb{X}$, these questions have been studied intensively before. For the classical heat kernel on $\mathbb{X}$, precise upper and lower bounds were obtained in [1] using spherical analysis,

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