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Monic representations of the Cuntz algebra and Markov measures

Dorin Ervin Dutkay [a,*], Palle E.T. Jorgensen [b]

[a] *University of Central Florida, Department of Mathematics, 4000 Central Florida Blvd., P.O. Box 161364, Orlando, FL 32816-1364, USA*

[b] *University of Iowa, Department of Mathematics, 14 MacLean Hall, Iowa City, IA 52242-1419, USA*

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We study representations of the Cuntz algebras $\mathcal{O}_N$. While, for fixed N, the set of equivalence classes of representations of $\mathcal{O}_N$ is known not to have a Borel cross section, there are various subclasses of representations which can be classified. We study monic representations of $\mathcal{O}_N$, that have a cyclic vector for the canonical abelian subalgebra. We show that $\mathcal{O}_N$ has a certain universal representation which contains all positive monic representations. A large class of examples of monic representations is based on Markov measures. We classify them and as a consequence we obtain that different parameters yield mutually singular Markov measure, extending the classical result of Kakutani. The monic representations based on the Kakutani measures are exactly the ones that have a one-dimensional cyclic S_i^*-invariant space.

* Corresponding author.
E-mail addresses: Dorin.Dutkay@ucf.edu (D.E. Dutkay), palle-jorgensen@uiowa.edu (P.E.T. Jorgensen).

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Contents

1. Introduction

The Cuntz algebra $\mathcal{O}_N$ is indexed by an integer $N > 1$, where N is the number of generators. As a C^*-algebra (denoted $\mathcal{O}_N$), it is defined by its generators and relations (the Cuntz-relations), and $\mathcal{O}_N$ is known to be a simple, purely infinite C^*-algebra, [8]. Further its K-groups are known. But its irreducible representations are highly subtle. To appreciate the importance of the study of representations of $\mathcal{O}_N$, recall that to specify a representation of $\mathcal{O}_N$ amounts to identifying a system of isometries in a Hilbert space $\mathcal{H}$, with mutually orthogonal ranges, and adding up to $\mathcal{H}$. But such orthogonal splitting in Hilbert space may be continued iteratively, and, as a result, one gets links between the study of $\mathcal{O}_N$-representation on the one hand, to such neighboring areas as symbolic dynamics and to filters used in signal processing, corresponding to a system of N uncorrelated frequency bands.

Returning to the subtleties of the representations of $\mathcal{O}_N$, and their equivalence classes, it is known that, for fixed N, that the set of equivalence classes of irreducible representations of $\mathcal{O}_N$, does not admit a Borel cross section; i.e., the equivalence classes, under unitary equivalence, does not admit a parameterization in the measurable Borel category. (Intuitively, they defy classification). Nonetheless, special families of inequivalent representations have been found, and they have a multitude of applications, both to mathematical physics [4], to the study of wavelets [16,15,23,22], to harmonic analysis [33,10,14], to the study of fractals as iterated function systems [12,17]; and to the study of $\mathrm{End}(B(\mathcal{H}))$ (= endomorphisms) where $\mathcal{H}$ is a fixed Hilbert space. Hence it is of interest to identify both discrete and continuous series of representations of $\mathcal{O}_N$, as they arise in such applications.

From Definition 2.1, it is evident that the problem of finding representations of $\mathcal{O}_N$, in a Hilbert space, and their properties, is a rather abstract one, and daunting. Unless the problem is first pared down and structured, there is little one can do in the way of finding and classify $\mathcal{O}_N$-representations. There is even a theorem of Glimm [19,20] to the effect all representations do not admit a Borel labeling; more precisely the set of equivalence classes of representations of $\mathcal{O}_N$ do not have a Borel cross section. Nonetheless