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On the membership of Hankel operators in a class of Lorentz ideals

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A B S T R A C T

Recall that the Lorentz ideal $\mathcal{C}_p^-$ is the collection of operators A satisfying the condition $\|A\|_p^- = \sum_{j=1}^{\infty} j^{-(p-1)/p} s_j(A) < \infty$. Consider Hankel operators $H_f : H^2(S) \to L^2(S, d\sigma) \ominus H^2(S)$, where $H^2(S)$ is the Hardy space on the unit sphere S in $\mathbf{C}^n$. In this paper we characterize the membership $H_f \in \mathcal{C}_p^-$, $2n < p < \infty$.

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1. Introduction

The study of Hankel operators has a long and rich history [1,2,4–7,10–14,17–19]. We are particularly interested in one kind of Hankel operators: those on the Hardy space of the unit sphere. Let us begin by describing our basic setting.

Let S be the unit sphere $\{z : |z| = 1\}$ in $\mathbf{C}^n$. In this paper, the complex dimension n is always assumed to be greater than or equal to 2. Let $d\sigma$ be the standard spherical measure on S. That is, $d\sigma$ is the positive, regular Borel measure on S with $\sigma(S) = 1$ that is invariant under the orthogonal group $O(2n)$, i.e., the group of isometries on $\mathbf{C}^n \cong \mathbf{R}^{2n}$ which fix 0.

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Recall that the Hardy space $H^2(S)$ is the norm closure in $L^2(S, d\sigma)$ of the collection of polynomials in the complex variables $z_1, \dots, z_n$. As usual, we let P denote the orthogonal projection from $L^2(S, d\sigma)$ onto $H^2(S)$. The main object of study in this paper, the Hankel operator $H_f : H^2(S) \to L^2(S, d\sigma) \ominus H^2(S)$, is defined by the formula

$$H_f = (1 - P)M_f|H^2(S).$$

To motivate what we will do in this paper, let us briefly review what has been done so far.

We consider symbol functions $f \in L^2(S, d\sigma)$. Recall that the problems of boundedness and compactness of H_f were settled in [17]. Later, in [5] we characterized the membership of H_f in the Schatten class $\mathcal{C}_p$, $2n < p < \infty$. Moreover, it was shown in [5] that the membership $H_f \in \mathcal{C}_{2n}$ implies $H_f = 0$. More recently, in [6] we characterized the membership of H_f in the ideal $\mathcal{C}_p^+$, $2n < p < \infty$.

In this paper, we turn our attention to the membership of H_f in the Lorentz ideal $\mathcal{C}_p^-$. Before going any further, it is necessary to recall the definition of these operator ideals.

Given an operator A, we write $s_1(A), \dots, s_j(A), \dots$ for its s-numbers [9, Section II.2]. For each $1 < p < \infty$, the formula

$$\|A\|_p^- = \sum_{j=1}^{\infty} \frac{s_j(A)}{j^{(p-1)/p}}$$

defines a symmetric norm for operators [9, Section III.15]. On any separable Hilbert space $\mathcal{H}$, the set

$$\mathcal{C}_p^- = \{A \in \mathcal{B}(\mathcal{H}) : \|A\|_p^- < \infty\}$$

is a norm ideal [9, Section III.2].

Closely associated with the Lorentz ideals $\mathcal{C}_p^-$ are the ideals $\mathcal{C}_p^+$, which are defined as follows. For each $1 \le p < \infty$, the formula

$$\|A\|_p^+ = \sup_{j \ge 1} \frac{s_1(A) + s_2(A) + \dots + s_j(A)}{1^{-1/p} + 2^{-1/p} + \dots + j^{-1/p}}$$

also defines a symmetric norm for operators [9, Section III.14]. On any separable Hilbert space $\mathcal{H}$, we have the norm ideal

$$\mathcal{C}_p^+ = \{A \in \mathcal{B}(\mathcal{H}) : \|A\|_p^+ < \infty\}.$$

As we mentioned, the $\mathcal{C}_p^+$'s were the ideals of interest in [6]. In this paper, these ideals will play an important supporting role.

Compared with the more familiar Schatten class $\mathcal{C}_p = \{A \in \mathcal{B}(\mathcal{H}) : \|A\|_p < \infty\}$, where $\|A\|_p = \{\operatorname{tr}((A^*A)^{p/2})\}^{1/2}$, for all $1 < p' < p < \infty$ we have the relation

$$\mathcal{C}_{p'}^+ \subset \mathcal{C}_p^- \subset \mathcal{C}_p \subset \mathcal{C}_p^+,$$

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