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Bloch functions on the unit ball of an infinite dimensional Hilbert space [☆]

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ABSTRACT

The Bloch space has been studied on the open unit disk of $\mathbb{C}$ and some homogeneous domains of $\mathbb{C}^n$. We define Bloch functions on the open unit ball of a Hilbert space E and prove that the corresponding space $\mathcal{B}(B_E)$ is invariant under composition with the automorphisms of the ball, leading to a norm that — modulo the constant functions — is automorphism invariant as well. All bounded analytic functions on B_E are also Bloch functions.

0. Introduction

The classical Bloch space $\mathcal{B}$ of analytic functions on the open unit disk $\mathbf{D}$ of $\mathbb{C}$ plays an important role in geometric function theory and it has been studied by many authors.

R.M. Timoney ([4] and [5]) extended the notion of Bloch function by considering bounded homogeneous domains in $\mathbb{C}^n$, such as the unit ball B_n and the polydisk $\mathbf{D}^n$.

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In this article, Bloch functions on the unit ball B_E of an infinite-dimensional Hilbert space E are introduced. We prove that a number results about Bloch functions on $\mathbf{D}$ and B_n can be extended to this infinite dimensional setting. Among them, several characterizations of Bloch functions on B_E known to hold in the finite dimensional case.

First, we will recall some background about the classical Bloch space and the space of Bloch functions on B_n. In Section 2, we will introduce the definition of $\mathcal{B}(B_E)$, the space of Bloch functions defined on B_E. A function $f : B_E \to \mathbb{C}$ is said to be a Bloch function if

$$\sup_{x \in B_E} \left(1 - \|x\|^2\right) \left\|\nabla f(x)\right\| < \infty.$$

Section 2 is devoted to the connection between functions in $\mathcal{B}(B_E)$ and their restrictions to one-dimensional subspaces seen as functions defined on $\mathbf{D}$ or either to their restrictions to finite-dimensional ones, resulting the fact that if for a given n, the restrictions of the function to the n-dimensional subspaces have their Bloch norms uniformly bounded, then the function is a Bloch one and conversely. We also introduce an equivalent norm for $\mathcal{B}(B_E)$ obtained by replacing the gradient by the radial derivative. We exhibit in Section 3 another equivalent norm for $\mathcal{B}(B_E)$ which is invariant — modulo the constant functions — under the action of the automorphisms of the ball. This is achieved without appealing to the invariant Laplacian and relying only on properties of automorphisms of B_E. Further, we are able to show that the space $H^\infty(B_E)$ of bounded analytic functions is contractively embedded in $\mathcal{B}(B_E)$, as it occurs in the finite dimensional case. Examples of unbounded Bloch functions are also shown.

1. Background

1.1. The classical Bloch space $\mathcal{B}$

The classical *Bloch space* $\mathcal{B}$ (see [3]) is the space of analytic functions $f : \mathbf{D} \longrightarrow \mathbb{C}$ satisfying

$$\|f\|_{\mathcal{B}} = \sup_{z \in \mathbf{D}} \left(1 - |z|^2\right) \left|f'(z)\right| < \infty$$

endowed with the norm

$$\|f\|_{Bloch} = \left|f(0)\right| + \|f\|_{\mathcal{B}} < \infty$$

so that $(\mathcal{B}, \|\cdot\|_{Bloch})$ becomes a Banach space.

It is well-known that the semi-norm $\|\cdot\|_{\mathcal{B}}$ is invariant by automorphisms, that is, $\|f \circ \varphi\|_{\mathcal{B}} = \|f\|_{\mathcal{B}}$ for any $f \in \mathcal{B}$ and $\varphi \in Aut(\mathbf{D})$. The following basic result can be proved applying Schwarz's lemma (see for instance [7]).