Journal of Functional Analysis 267 (2014) 1205–1240

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

Completely positive interpolations of compact, trace-class and Schatten-p class operators ☆

Ming-Hsiu Hsu [a], David Li-Wei Kuo [b], Ming-Cheng Tsai [b,*]

[a] Department of Mathematics, National Central University, Chung-Li 32054, Taiwan

[b] Department of Applied Mathematics, National Sun Yat-sen University, Kaohsiung, 80424, Taiwan

ARTICLE INFO

Article history:
Received 28 March 2014
Accepted 14 May 2014
Available online 27 May 2014
Communicated by G. Schechtman

MSC:
15B48
15B51
47L30
81P68

Keywords:
Completely positive linear maps
Trace-class
Compact normal operators
Schatten-p class operators

ABSTRACT

Extending Li and Poon's results on interpolation problems for matrices, we give characterizations of the existence of a completely positive linear map Φ_{cp} between compact (or Schatten-p class) operators sending a particular operator A to another B. It is shown that such a map exists if a multiple of the numerical range of A contains the numerical range of B. Given two commutative families of compact (or Schatten-p class) operators $\{A_\alpha\}$ and $\{B_\alpha\}$, we provide sufficient and necessary conditions to ensure that we can choose a completely positive interpolation Φ_{cp} to preserve trace and/or approximate units such that $\Phi_{\text{cp}}(A_\alpha) = B_\alpha$ for all α.

☆ The first author is supported by the Taiwan NSC Grant 102-2811-M-008-082. The third author is supported by the Taiwan NSC Grant 102-2811-M-110-018.

* Corresponding author.
E-mail addresses: hsumh@math.ncu.edu.tw (M.-H. Hsu), mpu.verilog@gmail.com (D.L.-W. Kuo), mctsai2@gmail.com (M.-C. Tsai).

http://dx.doi.org/10.1016/j.jfa.2014.05.013
0022-1236/

1. Introduction

Interpolation problems arise in many branches of study. Lagrange polynomial interpolation and Nevanlinna–Pick interpolation are two examples. In quantum information theory, people use compact positive operators with trace one to represent quantum states and trace-preserving completely positive linear maps to represent quantum channels (see [5,11,15]). One would like to ask the interpolation problem that whether there exists a quantum channel sending a particular input quantum state to a particular output quantum state. The corresponding problem for matrix algebras was considered by Li in [10,12]. In this paper, we relate the interpolation problem for compact normal operators to dilation theory and numerical ranges.

Given two operators A and B we try to answer the question of whether there exists a completely positive linear map ϕ such that $\phi(A) = B$. Let A be an $n \times n$ matrix and B an operator in $\mathcal{B}(H)$, where H is a separable Hilbert space which can be of finite or infinite dimension. Choi and Li [8] (see also [2–4]) show that there is a unital completely positive linear map $\Psi_{\mathrm{cp}} : M_n \to \mathcal{B}(H)$ such that

$$\Psi_{\mathrm{cp}}(A) = B, \tag{1.1}$$

if and only if there are a separable Hilbert space K and an isometry $V : H \to K$ such that

$$B = V^*(I \otimes A)V. \tag{1.2}$$

In this case, we set $\Psi_{\mathrm{cp}}(T) = V^*(I \otimes T)V$. When

$$A = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix},$$

one can deduce from Ando [1] (see also [7]) that (1.2) holds exactly when their numerical ranges satisfy

$$W(B) \subseteq W(A). \tag{1.3}$$

Here the numerical range $W(A)$ of an operator A in $\mathcal{B}(H)$ is defined by

$$W(A) = \big\{\langle Ax, x\rangle : \|x\| = 1\big\}.$$

When

$$A = \begin{pmatrix} \gamma_1 & 0 & 0 \\ 0 & \gamma_2 & 0 \\ 0 & 0 & \gamma_3 \end{pmatrix},$$