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Positive radial solutions for Dirichlet problems with mean curvature operators in Minkowski space ☆

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Abstract

In this paper, by using Leray–Schauder degree arguments and critical point theory for convex, lower semicontinuous perturbations of C^1-functionals, we obtain existence of classical positive radial solutions for Dirichlet problems of type

$$\operatorname{div}\left(\frac{\nabla v}{\sqrt{1-|\nabla v|^2}}\right)+f\left(|x|,v\right)=0 \quad \text{in } \mathcal{B}(R), \qquad v=0 \quad \text{on } \partial\mathcal{B}(R).$$

Here, $\mathcal{B}(R)=\{x\in\mathbb{R}^N\colon\ |x|<R\}$ and $f:[0,R]\times[0,\alpha)\to\mathbb{R}$ is a continuous function, which is positive on $(0,R]\times(0,\alpha)$.

Keywords: Dirichlet problem; Positive radial solutions; Mean curvature operator; Minkowski space; Leray–Schauder degree; Szulkin's critical point theory

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1. Introduction

The aim of this paper is to present some existence results for positive radial solutions of the Dirichlet problem with mean curvature operator in the flat Minkowski space

$$\mathbb{L}^{N+1} := \{(x,t):\ x \in \mathbb{R}^N,\ t \in \mathbb{R}\}$$

endowed with the metric

$$\sum_{j=1}^{N}(dx_j)^2 - (dt)^2.$$

It is known (see [3,26]) that the study of spacelike submanifolds of codimension one in $\mathbb{L}^{N+1}$ with prescribed mean extrinsic curvature leads to Dirichlet problems of the type

$$\mathcal{M}v = H(x,v) \quad \text{in } \Omega, \qquad v = 0 \quad \text{on } \partial\Omega, \tag{1}$$

where

$$\mathcal{M}v = \operatorname{div}\left(\frac{\nabla v}{\sqrt{1-|\nabla v|^2}}\right),$$

Ω is a bounded domain in $\mathbb{R}^N$ and the nonlinearity $H : \Omega \times \mathbb{R} \to \mathbb{R}$ is continuous.

The starting point of this type of problems is the seminal paper [12] which deals with entire solutions of $\mathcal{M}v = 0$ (see also [1] for $N = 2$). The equation $\mathcal{M}v =$ constant is then analyzed in [31], while $\mathcal{M}v = f(v)$ with a general nonlinearity f is considered in [8]. On the other hand, motivated by the study of stationary surfaces in Minkowski space, in [25] the author consider the Neumann problem

$$\mathcal{M}v = \kappa v + \lambda \quad \text{in } \mathcal{B}(R), \qquad \partial_\nu v = \mu \quad \text{on } \partial\mathcal{B}(R),$$

where $\mathcal{B}(R) = \{x \in \mathbb{R}^N:\ |x| < R\}$, $\lambda \neq 0$, $\kappa > 0$, $\mu \in [0,1)$ and $N = 2$. More general sign-changing nonlinearities are studied in [5].

If H is bounded, then it has been shown by Bartnik and Simon [3] that (1) has at least one solution $u \in C^1(\Omega) \cap W^{2,2}(\Omega)$. Also, when Ω is a ball or an annulus in $\mathbb{R}^N$ and the nonlinearity H has a radial structure (no boundedness assumptions), then it has been proved in [4] that (1) has at least one classical radial solution. This can be seen as a “universal” existence result for the above problem in the radial case. On the other hand, in this context the existence of positive solutions has been scarcely explored in the related literature. The importance of this type of study becomes apparent in many practical situations, for instance, when the trivial solution is present.

In Section 2 we consider the Dirichlet problem

$$\mathcal{M}v + f(|x|, v) = 0 \quad \text{in } \mathcal{B}(R), \qquad v = 0 \quad \text{on } \partial\mathcal{B}(R), \tag{2}$$

where $\mathcal{B}(R) = \{x \in \mathbb{R}^N:\ |x| < R\}$ and $f : [0,R] \times [0,\alpha) \to \mathbb{R}$ is a continuous function, which is positive on $(0,R] \times (0,\alpha)$. We prove that (2) has at least one classical positive radial solution provided that f is superlinear at 0 with respect to $\phi(s) = s/\sqrt{1-s^2}$, that is