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An elementary proof of Strauss conjecture

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ABSTRACT

In this paper, we consider the Cauchy problem for semilinear wave equation in high dimensions. First we use a Morawetz energy estimate which is obtained by integrating on the light cone to get a weighted $L^2 - L^2$ estimate of the solution, and then give an elementary proof of the weighted Strichartz estimate in Georgiev et al. [3], hence the Strauss conjecture. We also obtain a variant of the weighted Strichartz estimates and give the sharp estimate of the lifespan for the semilinear wave equation with subcritical nonlinearity.

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1. Introduction

In this paper, we consider the following Cauchy problem for semilinear wave equations of the form

$$\begin{cases} \Box\phi(t,x) = F\big(\phi(t,x)\big), & (t,x) \in \mathbb{R}^+ \times \mathbb{R}^n, \\ t = 0: \quad \phi = f(x),\ \phi_t = g(x),\ x \in \mathbb{R}^n, \end{cases} \tag{1.1}$$

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where

$$\Box = \partial_t^2 - \Delta, \tag{1.2}$$

and Δ is the Laplacian on $\mathbb{R}^n$. We assume that $f, g \in C_0^\infty(\mathbb{R}^n)$, and the nonlinear term F satisfies

$$F(0) = 0, \quad \left|F'(\phi)\right| \leq C|\phi|^{p-1}, \tag{1.3}$$

where $p > 1$ is a real number.

In 1979, John [7] showed that when $n = 3$ global solutions always exist if $p > 1 + \sqrt{2}$ and initial data are sufficiently small, and moreover, the global solutions do not exist if $F = |\phi|^p$, $p < 1 + \sqrt{2}$ and the initial data are not both identically zero. Generalizing John's results to other dimensions, Strauss conjectured that when $n \geq 2$ global solutions should always exist if initial data are sufficiently small and p is greater than a critical value $p_0(n)$ which is the positive root of the quadratic equation

$$\gamma(p, n) = -(n-1)p^2 + (n+1)p + 2 = 0. \tag{1.4}$$

This conjecture was verified when $n = 2$ by Glassey [4], $n = 4$ by the second author [19], $n \leq 8$ by Lindblad and Sogge [10] and finally for all $n \geq 4$ and $p \leq \frac{n+3}{n-1}$ by Georgiev, Lindblad and Sogge [3]. On the other hand, when $F = |\phi|^p$ and $1 < p < p_0(n)$ there can be blow-up for arbitrarily small data. This was shown by Glassey [5] when $n = 2$ and Sideris [13] for all $n \geq 4$. For the critical case $p = p_0(n)$ there still can be blow-up for small data, this was shown by Schaeffer [11] when $n = 2, 3$ and by Yordanov and Zhang [16] and independently by the second author [20] when $n \geq 4$.

The proof of Georgiev et al. [3] is based on a weighted Strichartz inequality for the inhomogeneous wave equation. Like the usual Strichartz estimates, it is interpolated from $L^1 - L^\infty$ estimate and $L^2 - L^2$ estimate. However in the latter case, one needs a weighted $L^2 - L^2$ estimate, which was proved by techniques from degenerated Fourier integral operators and is very difficult. Later, Tataru [15] gave a simple proof by changing the wave equation to that of a hyperbolic space. However, Tataru's proof is not elementary, it is based on Harmonic analysis on hyperbolic space. In this paper we observe that the $L^2 - L^2$ estimate of Georgiev et al. [3] can in fact be proved by a Morawetz energy estimate, which is obtained by integrating on the light cone. Thus, we give an elementary and simple proof of Strauss Conjecture with supercritical nonlinear term in high dimensions. Furthermore we obtain a variant of weighted Strichartz inequality. This enables us to get the sharp estimate of the lifespan for the solution of the problem with subcritical nonlinear term in high dimensions. The problem concerned with such lifespan estimate has been solved by various authors in low dimensional case, $n = 2$ was obtained by the second author [18], $n = 3$ was obtained by Lindblad [8] or the second author [17] independently. However to our knowledge, in high dimensions the problem in subcritical case has been open for more than 30 years, and neither the method of Georgiev et al. [3] nor the method of

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