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Moment formulae for general point processes

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The goal of this paper is to generalize most of the moment formulae obtained in [12]. More precisely, we consider a general point process μ, and show that the quantities relevant to our problem are the so-called Papangelou intensities. When the Papangelou intensities of μ are well-defined, we show some general formulae to recover the moment of order n of the stochastic integral of the point process. We will use these extended results to introduce a divergence operator and study a random transformation of the point process.

1. Introduction

Point processes constitute a general framework used to model a wide variety of phenomena. The underlying theory is well understood, and the relevant literature is

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abundant (see [2] and [6] for example). However, we have a much deeper understanding of the Poisson point process, which is one of the reasons for its use in a lot of practical cases. In particular, one has a chaos-expansion of Poisson functionals, concentration inequalities, moment formulae, etc. [12,11]. On the other hand, one lacks most of these tools for more general point processes. Our goal is to obtain moment formulae for very general point processes that are only required to have a Papangelou intensity. Intuitively, the Papangelou intensity c is such that $c(x,\xi)\,\lambda(\mathrm{d}x)$ is the conditional probability of finding a particle in the vicinity of x, given the configuration ξ (here λ is the reference measure). Historically, the first type of processes satisfying this condition is the Gibbs process. For a Gibbs process, $c(x,\xi) = e^{H(x\cup\xi)-H(\xi)}$, where H is a global energy function, chosen in a suitable class of functions. The interested reader can find further information regarding Gibbs processes in [14,15] as well as in [10]. More recently, determinantal processes have been found to have Papangelou intensities under certain conditions (see [4] as well as [19]).

As we have stated previously, our aim is to obtain moment formulae for quite general point processes. Hence, we follow the path of [13], in which some moment formulae were obtained for the Poisson point process (PPP). In particular, the main result of [13] is the following formula, obtained for the PPP:

$$\mathbb{E}\left[\left(\int u_y(\xi)\,\xi(\mathrm{d}y)\right)^n\right] = \sum_{\{P_1,\dots,P_k\}\in\mathcal{T}_n} \mathbb{E}\left[\int_{E^k} u_{x_1}^{|P_1|}\dots u_{x_k}^{|P_k|}(\xi\cup x_1\cup\dots\cup x_k)\,\lambda(\mathrm{d}x_1)\dots\lambda(\mathrm{d}x_k)\right],$$

where $\mathcal{T}_n$ is the set of all partitions of $\{1,\dots,n\}$, $|P_i|$ is the cardinality of P_i, $i=1,\dots,k$, and $u : E\times\mathcal{X}\to\mathbb{R}$ is a nonnegative measurable process.

The proofs in [13] are mostly based on the use of previous results related to Malliavin calculus (in particular the formula that gives $\mathbb{E}[\delta(u)^n]$). In this paper, we generalize all the formulae in [13] to the case of a point process which has Papangelou intensities (which obviously includes the case of the PPP). Our proofs are mainly based on the Georgii–Nguyen–Zessin formula, and as a consequence, we also obtain analogues of the formula that gives $\mathbb{E}[\delta(u)^n]$, for a suitable definition of $\delta(u)$ in our context, which was introduced in [18]. In all our results, the difference between the PPP and a general point process is a randomization of the underlying measure λ obtained by multiplying it by $c(\cdot,\xi)$.

Our results also allow us to study random transformations of point processes. In the case of a general point process μ, we consider a random transformation τ, such that each particle x of the configuration ξ is moved to $\tau(x,\xi)$. Then, we obtain an explicit characterization of $\tau^*\mu$ if we assume that τ satisfies a suitable condition, where here τ^* is the image measure of μ. An application is to show that a non-random transformation of a determinantal point process yields another determinantal point process.