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Multiple positive radial solutions for a Dirichlet problem involving the mean curvature operator in Minkowski space ☆

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Abstract

We study the Dirichlet problem with mean curvature operator in Minkowski space

$$\operatorname{div}\left(\frac{\nabla v}{\sqrt{1-|\nabla v|^2}}\right)+\lambda\left[\mu\left(|x|\right)v^q\right]=0 \quad \text{in } \mathcal{B}(R), \qquad v=0 \quad \text{on } \partial\mathcal{B}(R),$$

where $\lambda > 0$ is a parameter, $q > 1$, $R > 0$, $\mu : [0, \infty) \to \mathbb{R}$ is continuous, strictly positive on $(0, \infty)$ and $\mathcal{B}(R) = \{x \in \mathbb{R}^N\colon |x| < R\}$. Using upper and lower solutions and Leray–Schauder degree type arguments, we prove that there exists $\Lambda > 0$ such that the problem has zero, at least one or at least two positive radial solutions according to $\lambda \in (0, \Lambda)$, $\lambda = \Lambda$ or $\lambda > \Lambda$. Moreover, Λ is strictly decreasing with respect to R.

Keywords: Dirichlet problem; Positive radial solutions; Mean curvature operator; Minkowski space; Leray–Schauder degree; Upper and lower solutions

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1. Introduction

In this paper we present some non-existence, existence and multiplicity results for radial solutions of Dirichlet problems in a ball, associated to the mean curvature operator in the flat Minkowski space

$$\mathbb{L}^{N+1} := \{(x,t)\colon\ x \in \mathbb{R}^N,\ t \in \mathbb{R}\}$$

endowed with the Lorentzian metric

$$\sum_{j=1}^{N}(dx_j)^2 - (dt)^2,$$

where (x,t) are the canonical coordinates in $\mathbb{R}^{N+1}$.

These problems are originated in the study – in differential geometry or special relativity, of maximal or constant mean curvature hypersurfaces, i.e., spacelike submanifolds of codimension one in $\mathbb{L}^{N+1}$, having the property that their mean extrinsic curvature (trace of its second fundamental form) is respectively zero or constant (see e.g. [1,9,21]). More specifically, let M be a spacelike hypersurface of codimension one in $\mathbb{L}^{N+1}$ and assume that M is the graph of a smooth function $v : \Omega \to \mathbb{R}$ with Ω a domain in $\{(x,t)\colon\ x \in \mathbb{R}^N,\ t = 0\} \simeq \mathbb{R}^N$. The spacelike condition implies $|\nabla v| < 1$ and the mean curvature H satisfies the equation

$$\operatorname{div}\left(\frac{\nabla v}{\sqrt{1-|\nabla v|^2}}\right) = NH(x,v) \quad \text{in } \Omega.$$

If H is bounded, then it has been shown in [3] that the above equation has at least one solution $u \in C^1(\Omega) \cap W^{2,2}(\Omega)$ with $u = 0$ on $\partial\Omega$.

In this paper we consider the Dirichlet boundary value problem

$$\operatorname{div}\left(\frac{\nabla v}{\sqrt{1-|\nabla v|^2}}\right) + \lambda\left[\mu(|x|)v^q\right] = 0 \quad \text{in } \mathcal{B}(R), \qquad v = 0 \quad \text{on } \partial\mathcal{B}(R), \tag{1}$$

where $\lambda > 0$ is a parameter, $q > 1$, $R > 0$, $\mu : [0,\infty) \to \mathbb{R}$ is continuous, strictly positive on $(0,\infty)$ and $\mathcal{B}(R) = \{x \in \mathbb{R}^N\colon\ |x| < R\}$.

Using a variational type argument, in [8] it is shown that if

$$(q+1)R^N < \lambda N \int_0^R r^{N-1}\mu(r)(R-r)^{q+1}\,dr,$$

then problem (1) has at least one positive classical radial solution. In particular, it is clear that the above condition is satisfied provided that λ is sufficiently large. On account of the main result of this paper (Theorem 1), this result becomes more precise. Namely, we prove (Corollary 1) that

- *there exists $\Lambda > 0$ such that* (1) *has zero, at least one or at least two positive classical radial solutions according to $\lambda \in (0,\Lambda)$, $\lambda = \Lambda$ or $\lambda > \Lambda$. Moreover, Λ is strictly decreasing with respect to R.*