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Preserving closedness of operators under summation

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A B S T R A C T

We give a sufficient condition for the sum of two closed operators to be closed. In particular, we study the sum of two sectorial operators with the sum of their sectoriality angles greater than π. We show that if one of the operators admits bounded H^∞-calculus and the resolvent of the other operator satisfies a boundedness condition stronger than the standard sectoriality, but weaker than the bounded imaginary powers property in the case of UMD spaces, then the sum is closed. We apply the result to the abstract parabolic problem and give a sufficient condition for L^p-maximal regularity.

1. Introduction

Let E be a Banach space and A, B be two closed linear operators in E with domains $\mathcal{D}(A)$ and $\mathcal{D}(B)$ respectively. We study the problem of whether the sum $A+B$ with domain $\mathcal{D}(A)\cap\mathcal{D}(B)$ is closed. An important application of the closedness of the sum is the L^p-maximal regularity property. Consider the Cauchy problem

$$\begin{cases} f'(t)+Af(t)=g(t), & t\in(0,\tau) \\ f(0)=0 \end{cases} \tag{1.1}$$

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in the E-valued L^p-space $L^p(0,\tau;E)$, where $p>1$ and $\tau>0$ are finite and $-A$ is the infinitesimal generator of a bounded analytic semigroup on E. The operator A satisfies L^p*-maximal regularity* if for some p (and hence by [7] for all) we have that for any $g\in L^p(0,\tau;E)$ the unique solution

$$f(t)=\int_0^t e^{(x-t)A}g(x)\,dx$$

belongs to the first Sobolev space $W^{1,p}(0,\tau;E)$. It is not difficult to see that the above definition is independent of τ. For applications of the L^p-maximal regularity property to nonlinear problems, using Banach fixed point argument, we refer to [2].

In [8] the abstract problem of the closedness of the sum has been studied in the case of a UMD space (unconditionality of martingale differences) for sectorial operators that have bounded imaginary powers, and a sufficient condition for closedness has been given. In [10] the same problem has been examined in the approach of operator valued functional calculus, and it has been shown that R-sectoriality for one of the operators together with bounded H^∞-calculus for the other operator are sufficient for solution. In both cases, an application to the problem of L^p-maximal regularity has been given.

In this paper we study the closedness of the sum of two closed operators in the case of sectorial operators with the sum of their sectoriality angles greater than π. By using the classical formula for the inverse of the closure of the sum of the two operators and the ideas of Theorem 4.4 in [10] (i.e. dyadic decomposition of $\mathbb{R}$ and an appropriate use of the bounded H^∞-calculus property), we observe that the bounded imaginary powers property generates Banach space valued trigonometric polynomials and hence, an analogue boundedness condition can be imposed. In this way we show that if one of the operators has bounded H^∞-calculus and the resolvent of the other operator satisfies some Hardy–Littlewood majorant type inequality, which is a boundedness condition stronger than the standard sectoriality and similar to the R-sectoriality, then the sum is closed. In the case of spaces having UMD, we further show that the above boundedness condition is weaker than the bounded imaginary powers property. We finally apply the result to the problem (1.1) and give a sufficient condition for L^p-maximal regularity in the case of UMD spaces.

2. The closedness of $A+B$

The sectoriality property is essential since it implies closability for the sum of two closed operators.

Definition 2.1. Let E be a Banach space, $K\geqslant 1$ and $\theta\in[0,\pi)$. Let $\mathcal{P}_K(\theta)$ be the class of closed densely defined linear operators in E such that if $A\in\mathcal{P}_K(\theta)$, then

$$\Lambda_\theta=\{z\in\mathbb{C}\mid |\arg z|\leqslant\theta\}\cup\{0\}\subset\rho(-A)\quad\text{and}\quad (1+|z|)\|(A+z)^{-1}\|\leqslant K,\quad\forall z\in\Lambda_\theta.$$

Also, let $\mathcal{P}(\theta)=\bigcup_K\mathcal{P}_K(\theta)$. The elements in $\mathcal{P}(\theta)$ are called *sectorial operators of angle* θ.