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Central limit theorems for supercritical branching Markov processes

Yan-Xia Ren [a,1], Renming Song [b,2], Rui Zhang [c,*,3]

[a] *LMAM School of Mathematical Sciences & Center for Statistical Science, Peking University, Beijing, 100871, PR China*

[b] *Department of Mathematics, University of Illinois, Urbana, IL 61801, USA*

[c] *LMAM School of Mathematical Sciences, Peking University, Beijing, 100871, PR China*

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Abstract

In this paper we establish spatial central limit theorems for a large class of supercritical branching Markov processes with general spatial-dependent branching mechanisms. These are generalizations of the spatial central limit theorems proved in [1] for branching OU processes with binary branching mechanisms. Compared with the results of [1], our central limit theorems are more satisfactory in the sense that the normal random variables in our theorems are non-degenerate.

Keywords: Central limit theorem; Branching Markov process; Supercritical; Martingale; Eigenfunction expansion

1. Introduction

In recent years, there have been many papers on law of large numbers type convergence theorems for branching Markov processes and superprocesses, see, for instance, [10,11,16–18,

* Corresponding author.

E-mail addresses: yxren@math.pku.edu.cn (Y.-X. Ren), rsong@math.uiuc.edu (R. Song), ruizhang8197@gmail.com (R. Zhang).

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26,30,38] and the references therein. For recent results on other non-central limit theorem types convergence results for branching Markov processes, see, for instance, [19,20,28,29] and the references therein.

The focus of this paper is on spatial central limit theorems for branching Markov processes. For critical branching Markov processes starting from a Poisson random field or an equilibrium distribution, and subcritical branching Markov processes with immigration, some functional central limit theorems of the occupation times were established in a series of papers, see, for instance, [7–9,31–34] and reference therein. However, up to now, no spatial central limit theorems have been established for $\langle f, X_t\rangle$ of general supercritical branching Markov processes starting from general initial configurations. In [1], some spatial central limit theorems were established for $\langle f, X_t\rangle$ of supercritical branching OU processes with binary branching mechanism starting from a point mass. In [35], some spatial central limit theorems were established for supercritical super-OU processes with binary branching mechanisms starting from finite and compactly supported measures. However, the central limit theorems of [1,35] are not very satisfactory since the limiting normal random variables maybe degenerate. In the recent paper [36], we established spatial central limit theorems for supercritical super-OU processes with general branching mechanisms starting from finite and compactly supported measures. The limiting normal random variables in our central limit theorems are non-degenerate. For earlier central limit theorems for supercritical branching processes and supercritical multi-type branching processes, see [2–5,22,23].

In this paper, we will extend the arguments of [1,35,36] to establish spatial central limit theorems for a large class of supercritical branching Markov processes with general spatial-dependent branching mechanisms.

1.1. Spatial process

In this subsection, we spell out our assumptions on the spatial Markov process and then give some examples.

Suppose that E is a locally compact separable metric space and that μ is a σ-finite Borel measure on E with full support. Suppose that ∂ is a separate point not contained in E. ∂ will be interpreted as the cemetery point. We will use E_∂ to denote $E\cup\{\partial\}$. Every function f on E is automatically extended to E_∂ by setting $f(\partial)=0$. We will assume that $\xi=\{\xi_t, \Pi_x\}$ is a μ-symmetric Hunt process on E and $\zeta:=\inf\{t>0\colon \xi_t=\partial\}$ is the lifetime of ξ. We will use $\{P_t\colon t\geqslant 0\}$ to denote the semigroup of ξ. Our standing assumption on ξ is that there exists a family of continuous strictly positive symmetric functions $\{p_t(x,y)\colon t>0\}$ on $E\times E$ such that

$$P_t f(x)=\int_E p_t(x,y)f(y)\mu(dy).$$

It is well known and easy to check that, for $p\geqslant 1$, $\{P_t\colon t\geqslant 0\}$ is a strongly continuous contraction semigroup on $L^p(E,\mu)$. In fact, it follows from Hölder's inequality, Fubini's theorem and symmetry that

$$\begin{aligned}\|P_t f\|_p^p&=\int_E\left|\int_E p_t(x,y)f(y)\mu(dy)\right|^p\mu(dx)\\&\leqslant\int_E\int_E p_t(x,y)|f|^p(y)\mu(dy)\mu(dx)\leqslant\|f\|_p^p.\end{aligned}$$