



Complex classical fields: An example

Arthur Jaffe ^{a,*}, Christian D. Jäkel ^b, Roberto E. Martinez II ^a

^a *Harvard University, Cambridge, MA 02138, United States*

^b *School of Mathematics, Cardiff University, Wales, United Kingdom*

Received 14 August 2013; accepted 31 August 2013

Available online 8 October 2013

Communicated by S. Vaes

Dedicated to Arthur Strong Wightman¹

Abstract

We study complex, classical, scalar fields within a new framework introduced in a previous work. We replace the usual functional integral by a complex functional arising from a boosted Hamiltonian. We generalize the Feynman–Kac relation to this setting, and use it to establish the spectral condition on a cylinder. We consider also positive-temperature states.

© 2013 Elsevier Inc. All rights reserved.

Keywords: Constructive quantum field theory; Functional integral; Complex measure

Contents

1. Introduction	1834
2. Quantization	1835
2.1. Quantization of vectors	1836
2.2. Quantization of operators	1836
2.3. Quantization of time-zero fields	1837
2.4. The heat kernel semigroups	1837
2.5. Quantization domains	1838

* Corresponding author.

E-mail addresses: arthur_jaffe@harvard.edu (A. Jaffe), christian.jaekel@mac.com (C.D. Jäkel), remartin@fas.harvard.edu (R.E. Martinez).

¹ A.S. Wightman was an inspiration to the authors. We learned of his passing while finishing this work, which relates to topics that fascinated him.

3.	Classical Gaussian fields on \mathbb{R}^d	1839
3.1.	The two-point function $D_{\vec{v}}$	1839
3.2.	Time-reflection positivity	1843
3.3.	The classical Gaussian field	1844
3.4.	The Gaussian quantum field	1846
3.5.	Spatial reflection positivity	1847
3.6.	Quantization of spatial reflection positivity	1849
4.	Classical fields on the cylinder $X = \mathbb{R} \times \mathbb{T}^{d-1}$	1850
5.	A Feynman–Kac formula for $\mathcal{P}(\varphi)_2$ models	1851
5.1.	Background	1852
5.2.	Operators, forms, and the Feynman–Kac formula	1852
6.	The spectrum condition on the cylinder	1856
7.	Classical fields on the cylinder $X = S^1 \times \mathbb{R}^{d-1}$	1858
7.1.	Reflection positivity	1858
7.2.	Estimates on the kernels	1861
7.3.	Thermal quantization maps	1863
7.4.	Time translation and its unbounded quantization	1866
7.5.	The Tomita–Takesaki operators	1867
7.6.	The Araki–Woods Fock space	1868
7.7.	Quantization of field operators	1869
8.	Classical fields on the d -torus $X = \mathbb{T}^d$	1874
8.1.	The two-point function	1875
8.2.	Quantization	1875
9.	Some comments	1877
9.1.	Flat space and spatially compactified space	1878
9.2.	Compactified time	1879
	Acknowledgments	1880
	References	1880

1. Introduction

In [20] we introduced a framework for using complex classical fields to describe neutral, scalar quantum fields. In that work we replace the real functional integral by a complex functional. In this work we study the quantization of free complex fields in arbitrary spacetime dimension; in dimension two we also treat $\mathcal{P}(\varphi)_2$ -interactions on the spatial circle.

The mathematics of complex measures on finite-dimensional spaces poses no difficulty provided the absolute value of the measure can be integrated. The situation is more complicated for measures on function spaces, such as the measures in functional integrals. Not only can the density grow in certain complex directions, but also oscillations may lead to other difficulties with normalization. Even the case of Gaussian measures is not straightforward, so one can imagine more difficulty in the study of interactions with non-quadratic actions.

Complex fields arise naturally when the heat kernel of a Hamiltonian is complex, as in the case when an interaction breaks time-reversal symmetry. A simple family of examples arises when one adds a multiple of the momentum to the Hamiltonian. In this paper we consider perturbations of a Hamiltonian H with zero ground-state energy and with a positive heat kernel. We study perturbations of the form

$$H_{\vec{v}} = H + \vec{P} \cdot \vec{v}. \tag{1.1}$$

Download English Version:

<https://daneshyari.com/en/article/4590552>

Download Persian Version:

<https://daneshyari.com/article/4590552>

[Daneshyari.com](https://daneshyari.com)