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Complex classical fields: An example

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Abstract

We study complex, classical, scalar fields within a new framework introduced in a previous work. We replace the usual functional integral by a complex functional arising from a boosted Hamiltonian. We generalize the Feynman–Kac relation to this setting, and use it to establish the spectral condition on a cylinder. We consider also positive-temperature states.
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Keywords: Constructive quantum field theory; Functional integral; Complex measure

Contents

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[1] A.S. Wightman was an inspiration to the authors. We learned of his passing while finishing this work, which relates to topics that fascinated him.

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1. Introduction

In [20] we introduced a framework for using complex classical fields to describe neutral, scalar quantum fields. In that work we replace the real functional integral by a complex functional. In this work we study the quantization of free complex fields in arbitrary spacetime dimension; in dimension two we also treat $\mathscr{P}(\varphi)_2$-interactions on the spatial circle.

The mathematics of complex measures on finite-dimensional spaces poses no difficulty provided the absolute value of the measure can be integrated. The situation is more complicated for measures on function spaces, such as the measures in functional integrals. Not only can the density grow in certain complex directions, but also oscillations may lead to other difficulties with normalization. Even the case of Gaussian measures is not straightforward, so one can imagine more difficulty in the study of interactions with non-quadratic actions.

Complex fields arise naturally when the heat kernel of a Hamiltonian is complex, as in the case when an interaction breaks time-reversal symmetry. A simple family of examples arises when one adds a multiple of the momentum to the Hamiltonian. In this paper we consider perturbations of a Hamiltonian H with zero ground-state energy and with a positive heat kernel. We study perturbations of the form

$$H_{\vec{v}} = H + \vec{P} \cdot \vec{v}. \tag{1.1}$$

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