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Factoring multi-sublinear maps

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ABSTRACT

Using Rademacher type, maximal estimates are established for k-sublinear operators with values in the space of measurable functions. Maurey–Nikishin factorization implies that such operators factor through a weak-type Lebesgue space. This extends known results for sublinear operators and improves some results for bilinear operators. For example, any continuous bilinear operator from a product of type 2 spaces into the space of measurable functions factors through a Banach space. Also included are applications for multilinear translation invariant operators.

1. Introduction

Nikishin [23] showed that factorization through a weak-type Lebesgue space is equivalent to a particular maximal estimate as long as the underlying measure space is finite. Furthermore, using Rademacher type, Nikishin [23] showed that such an estimate holds for any sublinear operator which is continuous into the space of measurable functions. This result may be applied to the case of a σ-finite measure to obtain a factorization via multiplication by a positive measurable function and a change of density. Maurey [22] established an analogous characterization of factorization through strong-type Lebesgue spaces. Later, Pisier [28] proved a result intermediate to the those of Nikishin [23] and Maurey [22]. We will refer to these results as Maurey–Nikishin factorization. All of these

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characterizations equate vector-valued estimates with strong factorization, i.e. factorization given by a multiplication operator and a change of density.

In Section 1.1, we formulate our main result Theorem 1.1. The reduction to the finite measure case appears in Section 1.2 with some clarifying remarks about choosing an appropriate change of density.

Section 2 contains the proof of Theorem 1.1 by proving the corresponding k-sublinear maximal estimate, i.e. by proving Theorem 2.1. The analogous maximal estimate and corresponding factorization for Lebesgue space-valued operators appear in Section 2.1. These follow from similar arguments but use Pisier's [28] analog of Nikishin's [23] characterization of strong factorization.

In Section 3, Theorem 1.1 is combined with general results for bilinear operators appearing in [13] which identify additional geometric properties of the range of a bilinear operator. In particular, a bilinear operator from a product of type 2 spaces into the space of measurable functions must factor through a Banach space.

Section 4 contains applications to general translation invariant operators and convergence of multilinear Fourier series which generalize some results due to Stein [31]. Moreover, we borrow arguments from the proof of Maurey's [22] characterization of strong factorization to give a topological proof that non-trivial k-linear operators which commute with translations cannot map into certain Lebesgue spaces when the underlying group has infinite Haar measure. This provides a partial extension of a result due to Hörmander [8] for linear operators.

Section 5 contains closing remarks and open problems.

1.1. Preliminaries and main result

For each nonnegative integer j, let ϵ_j represent the j-th Rademacher function defined for $t \in [0,1]$. For $0 < p \leqslant 2$, a Banach space X has type p if there is a constant $T_p(X) < \infty$ such that

$$\left(\mathbb{E}\left\|\sum_j \epsilon_j x_j\right\|^2\right)^{1/2} \leqslant T_p(X)\left(\sum_j \|x_j\|^p\right)^{1/p} \tag{1.1}$$

for all n and $x_1,\dots,x_n \in X$. On the other hand, for $2 \leqslant q$, a Banach space X has cotype q if there is a constant $C_q(X) < \infty$ such that

$$\left(\sum_j \|x_j\|^q\right)^{1/q} \leqslant C_q(X)\left(\mathbb{E}\left\|\sum_j \epsilon_j x_j\right\|^2\right)^{1/2} \tag{1.2}$$

for all n and $x_1,\dots,x_n \in X$.

For a topological vector space X, a function $\|\cdot\| : X \to [0,\infty)$ is a quasi-norm if:

1. For all $x \in X$, $\|x\| \geqslant 0$ and $x = 0$ if and only if $\|x\| = 0$.