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Journal of Functional Analysis 266 (2014) 2225–2250

JOURNAL OF Functional Analysis

www.elsevier.com/locate/jfa

Confinement–deconfinement transitions for two-dimensional Dirac particles

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Received 24 May 2013; accepted 17 July 2013

Available online 31 July 2013

Communicated by L. Gross

Abstract

We consider a two-dimensional massless Dirac operator coupled to a magnetic field B and an electric potential V growing at infinity. We find a characterization of the spectrum of the resulting operator H in terms of the relation between B and V at infinity. In particular, we give a sharp condition for the discreteness of the spectrum of H beyond which we find dense pure point spectrum.

Keywords: Dirac operator; Graphene; Dense point spectrum

1. Introduction

Graphene, a two-dimensional lattice of carbon atoms arranged in a honeycomb structure, has attracted great attention in the last few years due to its unusual properties [1,13]. The dynamics of its low-energy excitations (the charge carriers) can be described by a two-dimensional massless Dirac operator D_0 [23,6], where the speed of light c is replaced by the Fermi velocity, $v_F \sim 10^{-2}c$. A remarkable property of these Dirac particles is their lack of localization in the presence of electric potential walls (i.e., potentials V with $V(\mathbf{x}) \to \infty$ as $|\mathbf{x}| \to \infty$). Indeed, if we assume

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0022-1236/$ – see front matter
http://dx.doi.org/10.1016/j.jfa.2013.07.018

that V is rotationally symmetric and of 'regular growth' the spectrum of the operator $D_0 + V$ equals the whole real line and is absolutely continuous [21,15,4,17]. It is also known, at least in three dimensions, that a much larger class of potentials growing at infinity do not produce eigenvalues [22,9].

One way to localize Dirac particles (in the sense that the Hamiltonian has non-trivial discrete spectrum) is through inhomogeneous magnetic fields when they are asymptotically constant [3, 11] as well as when they grow to infinity. Consider, for instance, a magnetic field $B = \operatorname{curl} \mathbf{A}$ with $B(\mathbf{x}) \to \infty$ as $|\mathbf{x}| \to \infty$ and denote by $D_{\mathbf{A}}$ the corresponding Dirac operator coupled to B. It is known that the spectrum of $D_{\mathbf{A}}$ is discrete away from zero and zero is an isolated eigenvalue of infinite multiplicity (see Section 3).

In this article we consider two-dimensional massless Dirac operators coupled to both an electric potential V and a magnetic field B. We study the combination of the two effects described above: The deconfinement effect associated to V and the confinement one associated to B.

Before presenting our main results let us first discuss this problem assuming that V and B are sufficiently regular positive rotationally symmetric functions. In this case, the Dirac operator admits an angular momentum decomposition $D_{\mathbf{A}} + V = \bigoplus_{j\in\mathbb{Z}} h_j$ and its spectrum, $\sigma(D_{\mathbf{A}} + V)$, satisfies

$$\sigma(D_{\mathbf{A}} + V) = \overline{\bigcup_{j\in\mathbb{Z}} \sigma(h_j)}.$$

Let $A(r) = \frac{1}{r}\int_0^r B(s)s\,\mathrm{d}s$ be the modulus of the magnetic vector potential in the rotational gauge. It is easy to show, on the one hand, that if $A(r) \to \infty$ as $r \to \infty$ and

$$\lim_{|\mathbf{x}|\to\infty} V\big(|\mathbf{x}|\big)/A\big(|\mathbf{x}|\big) < 1, \tag{1}$$

the spectrum of h_j is discrete, for each $j \in \mathbb{Z}$ (see Proposition 1 in Appendix A for the precise statement). On the other hand, as opposed to the non-relativistic case, it is known [18, Proposition 2] that if $V(r) \to \infty$ as $r \to \infty$ and

$$\lim_{|\mathbf{x}|\to\infty} V\big(|\mathbf{x}|\big)/A\big(|\mathbf{x}|\big) > 1, \tag{2}$$

the spectrum of each h_j equals the whole real line and is purely absolutely continuous. This phenomenon was recently discussed from the physical point of view in [7]. In that article a device was proposed to control the localization properties of particles in graphene by manipulating the electro-magnetic field at infinity, i.e., far away from the sample.

Clearly, if condition (2) is satisfied the spectrum of the full operator $H := D_{\mathbf{A}} + V$ is also absolutely continuous and equals $\mathbb{R}$. Conversely, one has pure point spectrum if condition (1) holds. It is, however, unclear whether the eigenvalues of H accumulate. Assume that $V(\mathbf{x}), B(\mathbf{x}) \to \infty$ as $|\mathbf{x}| \to \infty$. One may expect that when the quotient $|V(|\mathbf{x}|)/A(|\mathbf{x}|)|$ is sufficiently small, for large $|\mathbf{x}|$, the main effect of the electric potential is to remove the zero modes of $D_{\mathbf{A}}$ yielding purely discrete spectrum for H. However, as this quotient grows the eigenvalues of the h_j might accumulate creating points in the essential spectrum of H.

The aim of our work is to shed some light on the spectrum of H in terms of the relation between B and V at infinity. We emphasize that most of our results do not assume rotational