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The primitive ideals of the Cuntz–Krieger algebra of a row-finite higher-rank graph with no sources [☆]

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Abstract

We catalogue the primitive ideals of the Cuntz–Krieger algebra of a row-finite higher-rank graph with no sources. Each maximal tail in the vertex set has an abelian periodicity group of finite rank at most that of the graph; the primitive ideals in the Cuntz–Krieger algebra are indexed by pairs consisting of a maximal tail and a character of its periodicity group. The Cuntz–Krieger algebra is primitive if and only if the whole vertex set is a maximal tail and the graph is aperiodic.

Keywords: Higher-rank graph; k-Graph; Primitive ideal; $C(X)$-algebra; Irreducible representation

1. Introduction

Graph C^*-algebras were introduced in the early 1980's by Enomoto and Watatani [6] as an alternative description of the Cuntz–Krieger algebras invented in [4]. Enomoto and Watatani considered only finite graphs, but since the late 1990's substantial work has gone into describing and understanding the analogous construction for infinite directed graphs in various levels

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of generality (to name just a few, [2,8,10,14,18,23]). A gem in this program is Hong and Szymański's description [9] of the primitive ideal space of the C^*-algebra of a directed graph. Hong and Szymański catalogue the primitive ideals of $C^*(E)$ in terms of elementary structural features of E. They also describe the closure operation in the hull-kernel topology in terms of this catalogue.

In 1999, Robertson and Steger discovered a class of higher-rank Cuntz–Krieger algebras arising from $\mathbb{Z}^k$ actions on buildings [21]. Shortly afterwards, Kumjian and Pask introduced higher-rank graphs and the associated C^*-algebras [12] as a simultaneous generalisation of the graph C^*-algebras of [13] and Robertson and Steger's higher-rank Cuntz–Krieger algebras. Kumjian and Pask's k-graph algebras have since attracted a fair bit of attention. But their structure is more subtle and less well understood than that of graph C^*-algebras. In particular, while large portions of the gauge-invariant theory of graph C^*-algebras can be generalised readily to k-graphs, higher-rank analogues of other structure theorems for graph C^*-algebras are largely still elusive.

Our main result here is a complete catalogue of the primitive ideals in the C^*-algebra of a row-finite k-graph with no sources. Our methods are very different from Hong and Szymański's, and require a different set of technical tools. We have been unable to describe the hull-kernel topology on $\operatorname{Prim}(C^*(\Lambda))$, and we leave this question open for the present.

We begin in Section 2 by introducing P-graphs Γ and their C^*-algebras $C^*(\Gamma)$, where P is the image of $\mathbb{N}^k$ under a homomorphism f of $\mathbb{Z}^k$. We prove a gauge-invariant uniqueness theorem (Proposition 2.7) and a Cuntz–Krieger uniqueness theorem (Corollary 2.8) for these C^*-algebras. In Section 3, we consider the primitive ideal space of the pullback k-graph arising from a P-graph. We first show that if f and P are as above, then the pullback $f^*\Gamma$ of a P-graph Γ over $f : \mathbb{N}^k \to P$ and $d : \Gamma \to P$ is a k-graph. We then prove in Theorem 3.5 that the irreducible representations of $C^*(f^*\Gamma)$ are in bijection with pairs (π, χ) where π is an irreducible representation of $C^*(\Gamma)$, and χ is a character of $\ker f$.

In Section 4, we study maximal tails in k-graphs. Building on an idea from [5], we show that each maximal tail T has a well-defined *periodicity group* $\operatorname{Per}(T)$, and contains a large hereditary subset H such that the subgraph $H\Lambda T$ consisting of paths whose range and source both belong to H is isomorphic to a pullback of an $(\mathbb{N}^k/\operatorname{Per}(T))$-graph. The algebra $C^*(H\Lambda T)$ can be identified with a full corner in $C^*(\Lambda T)$. Combining this with our earlier results, we describe a bijection $(T, \chi) \mapsto I_{T,\chi}$ from pairs (T, χ) where T is a maximal tail of Λ and χ is a character $\operatorname{Per}(T)$ to primitive ideals of $C^*(\Lambda)$. We conclude by showing that $C^*(\Lambda)$ is primitive if and only if Λ^0 is a maximal tail and Λ is aperiodic.

As usual in our subject, our convention is that in the context of C^*-algebras, "homomorphism" always means "*-homomorphism", and "ideal" always means "closed two-sided ideal".

2. P-graphs

We introduce P-graphs over finitely generated cancellative abelian monoids P, and an associated class of C^*-algebras. Our treatment is very brief since these objects are introduced as a technical tool, and the ideas are essentially identical to those developed in [12]. Specialising to $P = \mathbb{N}^k$ in this section also serves to introduce the notation used later for k-graphs.

Definition 2.1. Let P be a finitely generated cancellative abelian monoid, which we also regard as a category with one object. A *P-graph* is a countable small category Γ equipped with a functor $d : \Gamma \to P$ which has the factorisation property: whenever $\xi \in \Gamma$ satisfies $d(\xi) = p + q$, there exist unique elements $\eta, \zeta \in \Gamma$ such that $d(\eta) = p$, $d(\zeta) = q$ and $\xi = \eta\zeta$.