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Corrigendum

Corrigendum to "The Conley conjecture for Hamiltonian systems on the cotangent bundle and its analogue for Lagrangian systems" [J. Funct. Anal. 256 (9) (2009) 2967–3034]

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Abstract

In lines 8–11 of Lu (2009) [18, p. 2977] we wrote: "For integer $m \geqslant 3$, if M is C^m-smooth and C^{m-1}-smooth $L : \mathbb{R} \times TM \to \mathbb{R}$ satisfies the assumptions (L1)–(L3), then the functional $\mathcal{L}_\tau$ is C^2-smooth, bounded below, satisfies the Palais–Smale condition, and all critical points of it have finite Morse indexes and nullities (see [1, Prop. 4.1, 4.2] and [4])". However, as proved in Abbondandolo and Schwarz (2009) [2] the claim that $\mathcal{L}_\tau$ is C^2-smooth is true if and only if for every (t, q) the function $v \mapsto L(t, q, v)$ is a polynomial of degree at most 2. So the arguments in Lu (2009) [18] are only valid for the physical Hamiltonian in (1.2) and corresponding Lagrangian therein. In this note we shall correct our arguments in Lu (2009) [18] with a new splitting lemma obtained in Lu (2011) [20].

Keywords: Conley conjecture; Hamiltonian and Lagrangian system; Cotangent and tangent bundle; Periodic solutions; Variational methods; Morse index; Maslov-type index

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1. A splitting lemma for C^1-functionals

In this section we shall give a special version of the splitting lemma obtained by the author in [20, Th. 2.1] recently. (The first splitting lemma was given by Gromoll and Meyer [11].) For completeness we shall outline its proof because it is much simpler than general case. The reader may refer to [20] for details.

Let H be a Hilbert space with inner product $(\cdot,\cdot)_H$ and the induced norm $\|\cdot\|$, and let X be a Banach space with norm $\|\cdot\|_X$, such that

(S) $X \subset H$ is dense in H and $\|x\| \leqslant \|x\|_X$ $\forall x \in X$.

For an open neighborhood V of the origin $\theta \in H$, $V \cap X$ is also an open neighborhood of θ in X, and we shall write $V \cap X$ as V_X when viewed as an open neighborhood of θ in X. For a C^1-functional $\mathcal{L}: V \to \mathbb{R}$ with θ as an isolated critical point, suppose that there exist maps $A \in C^1(V_X, X)$ and $B \in C(V_X, L_s(H))$ such that

$$\mathcal{L}'(x)(u) = \big(A(x), u\big)_H \quad \forall x \in V_X \text{ and } u \in X, \tag{1.1}$$

$$\big(A'(x)(u), v\big)_H = \big(B(x)u, v\big)_H \quad \forall x \in V_X \text{ and } u, v \in X. \tag{1.2}$$

(These imply: (a) $\mathcal{L}|_{V_X} \in C^2(V_X, \mathbb{R})$, (b) $d^2\mathcal{L}|_{V_X}(x)(u, v) = (B(x)u, v)_H$ for any $x \in V_X$ and $u, v \in X$, (c) $B(x)(X) \subset X$ $\forall x \in V_X$.) Furthermore we also assume B to satisfy the following properties:

(B1) If $u \in H$ such that $B(\theta)(u) = v$ for some $v \in X$, then $u \in X$. Moreover, all eigenfunctions of the operator $B(\theta)$ that correspond to negative eigenvalues belong to X.

(B2) The map $B: V_X \to L_s(H, H)$ has a decomposition

$$B(x) = P(x) + Q(x) \quad \forall x \in V \cap X,$$

where $P(x): H \to H$ is a positive definitive linear operator and $Q(x): H \to H$ is a compact linear operator with the following properties:

(i) For any sequence $\{x_k\} \subset V \cap X$ with $\|x_k\| \to 0$ it holds that $\|P(x_k)u - P(\theta)u\| \to 0$ for any $u \in H$;

(ii) The map $Q: V \cap X \to L(H, H)$ is continuous at θ with respect to the topology induced from H on $V \cap X$;

(iii) There exist positive constants $\eta_0 > 0$ and $C_0 > 0$ such that

$$\big(P(x)u, u\big) \geqslant C_0\|u\|^2 \quad \forall u \in H,\ \forall x \in B_H(\theta, \eta_0) \cap X.$$

Note. Since $B(\theta) \in L_s(H)$ is a self-adjoint Fredholm operator, either $0 \notin \sigma(B(\theta))$ or 0 is an isolated point in $\sigma(B(\theta))$ which is also an eigenvalue of finite multiplicity. See Proposition B.2 in Appendix of [20].