Available online at www.sciencedirect.com

SciVerse ScienceDirect

Journal of Functional Analysis 264 (2013) 1125–1129

JOURNAL OF Functional Analysis

www.elsevier.com/locate/jfa

Addendum

Addendum to “The limit equation for the Gross–Pitaevskii equations and S. Terracini’s conjecture” [J. Funct. Anal. 262 (3) (2012) 1087–1131] ☆

E.N. Dancer [a], Kelei Wang [a], Zhitao Zhang [b,*]

[a] *School of Mathematics and Statistics, University of Sydney, NSW 2006, Australia*

[b] *Academy of Mathematics and Systems Science, The Chinese Academy of Sciences, Beijing 100190, China*

Received 6 July 2012; accepted 15 October 2012

Available online 14 December 2012

Communicated by H. Brezis

Abstract

We study the differential systems arising from Bose–Einstein condensates under the assumption that all the diffusion constants d_i are not equal. We give interesting addenda to our previous papers: [J. Funct. Anal. 262 (3) (2012) 1087–1131] and [J. Differential Equations 251 (2011) 2737–2769].

Keywords: Gross–Pitaevskii equation; Singular perturbation; Free boundary problem; Partial regularity; Monotonicity formula

☆ Dancer and Wang were supported by the Australian Research Council, Zhang was supported by NSFC 10831005, 10971046, 11271353.

DOI of original article: http://dx.doi.org/10.1016/j.jfa.2011.10.013.

* Corresponding author.

E-mail addresses: normd@maths.usyd.edu.au (E.N. Dancer), kelei@maths.usyd.edu.au (K. Wang), zzt@math.ac.cn (Z. Zhang).

0022-1236/$ – see front matter
http://dx.doi.org/10.1016/j.jfa.2012.10.009

1. Main results

In the paper [4], for the system

$$\frac{\partial u_i}{\partial t} - d_i \Delta u_i = f_i(u_i) - \kappa u_i \sum_{j \neq i} b_{ij} u_j^2, \tag{1}$$

we proved the main theorem under the assumption that all the diffusion constants d_i are equal (for related papers, see [5,6] etc.). There we claimed that the proof is also valid in the general case. However this is not clear because the proof of the Almgren type monotonicity formula is unknown in the general case. Thus the result is unknown when d_i are not the same. Note that results for the elliptic problem are unaffected (by a rescaling).

The same difficulties with this monotonicity formula also affect the proof of the main result in [3]. However, this is not a real difficulty, because a more careful examination of the blow up analysis shows that we really only need the result for the blow up limit when there is only one species.

Assume a sequence of solutions of (1), $u_\kappa = (u_{1,\kappa}, u_{2,\kappa,\ldots})$ converges uniformly to $u = (u_1, 0, \ldots)$. Then (4.12) in [3] should be (here G is the heat kernel corresponding to d_1, and we refer to [3] for more notation)

$$\begin{aligned}
&D'_\kappa(t)\\
&= \int_{\mathbb{R}^n} \sum_i \left[\frac{x \cdot \nabla u_{i,\kappa}(x,-t)}{2d_1 t} - \frac{\partial u_{i,\kappa}}{\partial t}(x,-t)\right]^2 \varphi^2(x) G(x,t)\\
&\quad + \int_{\mathbb{R}^n} \sum_i \left(\frac{d_i}{d_1} - 1\right) \frac{x \cdot \nabla u_{i,\kappa}(x,-t)}{2d_1 t} \left[\frac{x \cdot \nabla u_{i,\kappa}(x,-t)}{2d_1 t} - \frac{\partial u_{i,\kappa}}{\partial t}(x,-t)\right] \varphi^2(x) G(x,t)\\
&\quad - \int_{\mathbb{R}^n} \sum_i 2d_i \left[\frac{x \cdot \nabla u_{i,\kappa}(x,-t)}{2d_1 t} - \frac{\partial u_{i,\kappa}}{\partial t}(x,-t)\right] \nabla u_{i,\kappa}(x,-t) \cdot \nabla \varphi(x) \varphi(x) G(x,t)\\
&\quad + \int_{\mathbb{R}^n} \left[\sum_i \frac{d_i}{2} \left|\nabla u_{i,\kappa}(x,-t)\right|^2 + F\big(u_\kappa(x,-t)\big) + H_\kappa\big(u_\kappa(x,-t)\big)\right] t^{-1} \varphi(x) x \cdot \nabla \varphi(x) G(x,t)\\
&\quad - \int_{\mathbb{R}^n} \sum_i \frac{d_i}{2} t^{-1} \left|\nabla u_{i,\kappa}(x,t)\right|^2 \varphi^2(x) G(x,t).
\end{aligned}$$

Compared with the case where d_i are equal, there appears a new term, the second term in the right-hand side. Because $\nabla u_{i,\kappa}$ converges to ∇u_i strongly in L^2_{loc} and $\frac{\partial u_{i,\kappa}}{\partial t}$ converges to $\frac{\partial u_i}{\partial t}$ weakly in L^2_{loc}, as $\kappa \to +\infty$, by our assumption on u, the second term in the right-hand side converges to 0 and we still get (4.13) in [3]. Following the proof in [3] it follows that the Almgren type monotonicity formula still holds if the singular limit $u = (u_1, u_2, \ldots)$ has only one component nonvanishing. Note that in this case we can avoid the argument in Section 5 of [3] and apply the main result in Section 10 of [4] to deduce that u_1 satisfies the heat equation everywhere and then use Liouville theorems for heat equations.