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On the conservativeness and the recurrence of symmetric jump-diffusions

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Abstract

Sufficient conditions for a symmetric jump-diffusion process to be conservative and recurrent are given in terms of the volume of the state space and the jump kernel of the process. A number of examples are presented to illustrate the optimality of these conditions; in particular, the situation is allowed to be that the state space is topologically disconnected but the particles can jump from a connected component to the other components.

Keywords: Regular Dirichlet form; Jump process; Integral-derivation property; Conservation property; Recurrence

Contents

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1. Introduction and main results

Let (X, d, m) be a metric measure space. We assume that every metric ball $B(x, r) = \{z \in X: d(x, z) < r\}$ centered at $x \in X$ with radius $r > 0$ is pre-compact, and the measure m is a Radon measure with full support. In particular, X is locally compact and separable. Let $(\mathcal{E}, \mathcal{F})$ be a regular symmetric Dirichlet form in $L^2(X; m)$. We denote the extended Dirichlet space of $(\mathcal{E}, \mathcal{F})$ by $\mathcal{F}_e$, and a quasi-continuous version of $u \in \mathcal{F}_e$ by $\tilde{u}$. According to the Beurling–Deny theorem, see, e.g., [8, Theorem 3.2.1 and Lemma 4.5.4], we can express $(\mathcal{E}, \mathcal{F})$ as follows

$$\mathcal{E}(u, v) = \mathcal{E}^{(c)}(u, v) + \iint_{x \neq y} \big(\tilde{u}(x) - \tilde{u}(y)\big)\big(\tilde{v}(x) - \tilde{v}(y)\big)\, J(dx, dy)$$
$$+ \int_X \tilde{u}(x)\tilde{v}(x)\, k(dx) \quad \text{for any } u, v \in \mathcal{F}_e,$$

where $(\mathcal{E}^{(c)}, C_0(X) \cap \mathcal{F})$ is a strongly-local symmetric form and $C_0(X)$ is the space of all real-valued continuous functions on X with compact support; J is a symmetric positive Radon measure on the product space $X \times X$ off the diagonal $\{(x, x): x \in X\}$; and k is a positive Radon measure on X.

Let $\mu_{\langle \cdot, \cdot \rangle}$ be a bounded signed measure, see [8, Lemma 3.2.3], such that

$$\mathcal{E}^{(c)}(u, v) = \frac{1}{2}\mu_{\langle u,v \rangle}(X) = \frac{1}{2}\int_X \mu_{\langle u,v \rangle}(dx) \quad \text{for } u, v \in \mathcal{F}_e.$$

Throughout the paper, we assume the following set (A) of conditions:

(A-1) The killing measure k does not appear; that is, the corresponding process is *no killing inside*.

(A-2) For each $u, v \in \mathcal{F}_e$, the measure $\mu_{\langle u,v \rangle}$ is absolutely continuous with respect to m. We denote the corresponding Radon–Nikodym density by $\Gamma^{(c)}(u, v)$; namely,

$$\mu_{\langle u,v \rangle}(dx) = \Gamma^{(c)}(u, v)(x)\, m(dx).$$

(A-3) The jump measure J has a symmetric kernel $j(x, dy)$ over $X \times \mathcal{B}(X)$ such that

$$J(dx, dy) = j(x, dy)\, m(dx)\big(= j(y, dx)\, m(dy) = J(dy, dx)\big).$$