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Existence and asymptotic stability of periodic solution for evolution equations with delays ☆

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Abstract

In this paper, we discuss the existence and asymptotic stability of the time periodic solution for the evolution equation with multiple delays in a Hilbert space H

$$u'(t) + Au(t) = F\big(t, u(t), u(t-\tau_1), \ldots, u(t-\tau_n)\big), \quad t \in \mathbb{R},$$

where $A : D(A) \subset H \to H$ is a positive definite selfadjoint operator, $F : \mathbb{R} \times H^{n+1} \to H$ is a nonlinear mapping which is ω-periodic in t, and $\tau_1, \tau_2, \ldots, \tau_n$ are positive constants. We present essential conditions on the nonlinearity F to guarantee that the equation has ω-periodic solutions or an asymptotically stable ω-periodic solution. The discussion is based on analytic semigroups theory and an integral inequality with delays.

Keywords: Evolution equations with delays; Time periodic solutions; Existence and uniqueness; Asymptotic stability; Analytic semigroups

1. Introduction and main results

The theory of partial differential equations with delays has extensive physical background and realistic mathematical model, hence it has been considerably developed and the numerous

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properties of their solutions have been studied, see [4,16] and references therein. The problems concerning periodic solutions of partial differential equations with delays are an important area of investigation in recent years. Specially, the existence of periodic solutions of evolution equations with delays has been considered by several authors, see [3,17,9–11,18]. In [3], Burton and Zhang researched an abstract evolution equation with infinite delay. Under the assumption that the solutions of the associated homotopy equations are uniformly bounded, they obtained the existence of periodic solutions by using Granas's fixed theorem. In [17], Xiang and Ahmed showed an existence result of periodic solution to the delay evolution equations in Banach spaces under the assumption that the corresponding initial value problem has a priori estimate. In [9–11], Liu derived periodic solutions from bounded solutions or ultimate bounded solutions for finite or infinite delay evolution equations in Banach spaces. In all these works, the key assumption of prior boundedness of solutions was employed.

Recently, Zhu, Liu and Li in [18] investigated the existence of time periodic solutions for the one-dimensional parabolic evolution equation with delays

$$\begin{cases} u_t = u_{xx} + au + f\big(u(x,t-\tau_1),\dots,u(x,t-\tau_n)\big) + g(x,t), & \text{in } (0,1)\times\mathbb{R}, \\ u(0,t) = u(1,t) = 0, & \text{in } \mathbb{R}, \end{cases} \tag{1}$$

where $a \in \mathbb{R}$, $f : \mathbb{R}^n \to R$ is locally Lipschitz continuous, $g : [0,1]\times\mathbb{R} \to \mathbb{R}$ is Hölder continuous and $g(x,t)$ is ω-periodic in t, and $\tau_1, \tau_2, \dots, \tau_n$ are positive constants. This equation models some process of biology, see [18,12]. Under the following assumptions

(A1) $n \leqslant 3$;
(A2) $|f(\eta_1,\dots,\eta_n) + g(x,t)| \leqslant \sum_{i=1}^n \beta_i|\eta_i| + K$ for $(\eta_1,\dots,\eta_n) \in \mathbb{R}^n$, where $\beta_1,\dots,\beta_n$ and K are positive constants;
(A3) $(|a|+2)^2 + \sum_{i=1}^n \beta_i^2 < \pi^2 + 1$;

they obtained the existence of time ω-periodic solutions to Eq. (1). Moreover, adding the condition

(A4) $|f(\eta_1,\dots,\eta_n) - f(\xi_1,\dots,\xi_n)| \leqslant \sum_{i=1}^n \beta_i|\eta_i - \xi_i|$,

they showed that the time periodic solutions of Eq. (1) are unique and asymptotically stable. The main steps of their arguments consist in constructing some suitable Lypunov functionals and establishing the prior bound for all possible periodic solutions.

In this paper, we will use a completely different method to improve and extend the results mentioned above. We will delete the condition (A1) and improve the condition (A3). We will use the concise condition

(A3)* $a + \sum_{i=1}^n \beta_i < \pi^2$,

instead of (A3). In fact, if the condition (A3) holds, then

$$\big(|a|+2\big)^2 < \pi^2 + 1, \qquad \sum_{i=1}^n \beta_i^2 < \pi^2 + 1 - \big(|a|+2\big)^2.$$