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Journal of Functional Analysis 262 (2012) 3409–3429

JOURNAL OF Functional Analysis

www.elsevier.com/locate/jfa

Global existence and blowup of a nonlocal problem in space with free boundary ☆

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Received 23 September 2011; accepted 19 January 2012

Available online 28 January 2012

Communicated by F.-H. Lin

Abstract

This paper concerns a double fronts free boundary problem for the reaction–diffusion equation with a nonlocal nonlinear reaction term in space. For such a problem, we mainly study the blowup property and global existence of the solutions. Our results show that if the initial value is sufficiently large, then the blowup occurs, while the global fast solution exists for a sufficiently small initial data, and the intermediate case with a suitably large initial data gives the existence of the global slow solution.

Keywords: Free boundary; Blowup; Global fast solution; Global slow solution

1. Introduction

As we know, there are several biological and physical phenomena that can be generally modeled by the following scalar differential equation:

$$u_t(t,x) = Eu(t,x) + f\big(x, u(t,x)\big),$$

☆ The work is partially supported by PRC grant NSFC 11071209, 10801115 and also by NSF 10KJB110011 of Jiangsu Province.

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0022-1236/$ – see front matter
doi:10.1016/j.jfa.2012.01.018

where E represents a linear elliptic operator of second order and x lies in a bounded domain in R^N. In this equation, the relation between the unknown function u and its derivatives is local in space in the sense that all functions are taken at the same point x.

However, there are some cases where a stronger, more global, spatial coupling occurs in the phenomena portrayed and has to be joined in the model. One feasible way of doing this is throughout an integrating over a region in space. Such models have a long history in mathematical literature and at least date back to Liouville who studied the equation

$$u_t = u_{xx} - b^2 x \int_0^1 x u_t \, \mathrm{d}x$$

relating to some problems in thermo-mechanics [26]. Since then, a number of other models of this type have been proposed and investigated. One of the most salient examples may be the original equation described as a model for turbulence in [2]:

$$\begin{cases} \dfrac{\mathrm{d}u}{\mathrm{d}t} = P - \dfrac{u}{R} - \displaystyle\int_0^1 v^2 \, \mathrm{d}x, \\ v_t + 2vv_x = \dfrac{1}{R} v_{xx} + uv, \end{cases}$$

where u denotes the velocity in a channel produced by some applied force P, while v is on behalf of the turbulent perturbation of the motion and R is the Reynolds constant associated with the viscosity of the fluid. Recently, there have been some literature devoted to the research of this type in the following form:

$$u_t - \Delta u = f(x, u, \bar{u}),$$

where $\bar{u} = \int_\Omega g(y, u(t, y)) \, \mathrm{d}y$ represents the type of strong coupling.

These problems have been applied in many areas, for example, obstacle problems [25], biological evolution [4], population dynamics [5,16], combustion theory [1], phase separation in binary mixtures [6,15,29] and so on. What deserves mentioning here is that there is another way in which nonlocal reaction term of an integral form in space may arise when some simplification is introduced in a local problem, and one can consult [20] for more details.

Recently the free boundary problem has attracted much attention in many areas, for example, the American option prizing problem [18,22], the wound healing [7], the combustion under gravity conditions [19], the tumor growth [9] and so on. Furthermore, the well-known Stefan condition has been used in the modeling of a number of applied problems. For example, it was used to describe the melting of ice in contact with water [28], the spreading of species [11,12, 21,24]. There is a vast literature on the Stefan problem, and some important theoretical advances can be found in [3,8].

In this paper, we will study the behavior of the positive solution $u(t, x)$ to the following free boundary problem with space integral term: