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Maximal averages over hypersurfaces and the Newton polyhedron ☆

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Abstract

Using some resolution of singularities and oscillatory integral methods in conjunction with appropriate damping and interpolation techniques, L^p boundedness theorems for $p > 2$ are obtained for maximal averages over hypersurfaces in R^n for $n > 2$. These estimates are sharp in various situations, including the convex hypersurfaces of finite line type considered by several authors. As a corollary, we also give a generalization of the result of Sogge and Stein that for some finite p the maximal operator corresponding to a hypersurface whose Gaussian curvature does not vanish to infinite order is bounded on L^p for some finite p. Analogous estimates are proven for Fourier transforms of surface measures, and these are sharp for the same hypersurfaces as the maximal operators.

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1. Introduction and statement of results

In this paper we are concerned with two closely related objects in analysis, maximal averages over hypersurfaces and Fourier transforms of surface-supported measures. Let S be a smooth hypersurface in $\mathbf{R}^{n+1}$ for $n \geqslant 2$, and let σ denote the standard surface measure on S. We consider

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the maximal operator, defined initially on Schwartz functions, given by

$$Mf(x) = \sup_{t>0} \left| \int_S f(x - ts)\phi(s)\, d\sigma(s) \right| \tag{1.1}$$

Here $\phi(x)$ is a smooth cutoff function that localizes the surface S near some specific $y \in S$. The goal here is to determine the values of p for which M is bounded on L^p. The earliest work on this subject was done in the case where S is a sphere, when Stein [23] showed M is bounded on L^p iff $p > \frac{n+1}{n}$ for $n > 1$. This was later generalized by Greenleaf [9] to surfaces of nonvanishing Gaussian curvature, and the $n = 1$ case was later proven by Bourgain [1]. Since then, there have been a wide range of papers on this subject, which we will describe in more detail throughout this section. Although there are many interesting issues when $p \leqslant 2$, for the purposes of this paper we always assume $p > 2$. Note that if M is bounded on some L^p, by interpolating with the L^∞ case one has that M is bounded on $L^{p'}$ for $p' > p$. Hence our goal is to determine the optimal $p_0 \geqslant 2$ for which M is bounded on L^p for $p > p_0$.

Let L be an invertible linear transformation, and let M_L be the maximal operator corresponding to the surface $L(S)$. Then one can easily check from the definitions that $M_L f(x) = |det(L)| M(f \circ L)(L^{-1}x)$. Hence in our arguments we may replace M by M_L at will. In particular, without loss of generality we henceforth assume that $(0, \dots, 0, 1)$ is not in the tangent plane $T_y(S)$. We do this so that we may represent S near y as the graph of some function $g(x_1, \dots, x_n)$, which permits us to do the coordinate-dependent analysis of this paper. Let z be such that $(z, g(z)) = y$ and define $h(x) = g(x) - g(z) - \nabla g(z) \cdot (x - z)$. Geometrically, $h(x)$ is the vertical distance from S to $T_y(S)$ over x. If $y \notin T_y(S)$ and $h(x)$ has a zero of infinite order at z, it is not hard to show that M is bounded on no L^p space for $p < \infty$ as long as ϕ is nonnegative with $\phi(y) \neq 0$. On the other hand, by a result of Sogge and Stein [22] if the Gaussian curvature of S does not vanish to infinite order at y then the reverse holds; M is bounded on L^p for some finite p as long as the support of ϕ is sufficiently small. (See also [5] for another theorem of this kind.) In Corollary 1.4 we will generalize this further.

The other (related) subject we are interested in this paper is the decay of Fourier transforms of surface measures. Let S and ϕ be as above and consider $T(\lambda)$ defined by

$$T(\lambda) = \int_S e^{-i\lambda \cdot x} \phi(x)\, d\sigma(x) \tag{1.2}$$

T may be recognized as the Fourier transform of the surface measure of S localized around y. As is well known, T is closely related to the maximal operator M. We may assume that the support of $\phi(x)$ is small enough such that we may write

$$T(\lambda) = \int e^{-i\lambda_1 x_1 - \dots - i\lambda_n x_n - i\lambda_{n+1} g(x_1, \dots, x_n)} \psi(x_1, \dots, x_n)\, dx_1 \cdots dx_n \tag{1.3}$$

Here $\psi(x_1, \dots, x_n)$ is now a cutoff function localized around the z such that $(z, g(z)) = y$. The main goal for $T(\lambda)$ is to determine the optimal $\epsilon > 0$ for which one has an estimate

$$\left|T(\lambda)\right| \leqslant C|\lambda|^{-\epsilon} \tag{1.4}$$

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