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Banach spaces with the Daugavet property, and the centralizer ☆

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Abstract

We introduce representable Banach spaces, and prove that the class $\mathcal{R}$ of such spaces satisfies the following properties:

(1) Every member of $\mathcal{R}$ has the Daugavet property.
(2) It Y is a member of $\mathcal{R}$, then, for every Banach space X, both the space $\mathcal{L}(X, Y)$ (of all bounded linear operators from X to Y) and the complete injective tensor product $X \widehat{\otimes}_\epsilon Y$ lie in $\mathcal{R}$.
(3) If K is a perfect compact Hausdorff topological space, then, for every Banach space Y, and for most vector space topologies τ on Y, the space $\mathcal{C}(K, (Y, \tau))$ (of all Y-valued τ-continuous functions on K) is a member of $\mathcal{R}$.
(4) If K is a perfect compact Hausdorff topological space, then, for every Banach space Y, most $\mathcal{C}(K, Y)$-superspaces (in the sense of [V. Kadets, N. Kalton, D. Werner, Remarks on rich subspaces of Banach spaces, Studia Math. 159 (2003) 195–206]) are members of $\mathcal{R}$.
(5) All dual Banach spaces without minimal M-summands are members of $\mathcal{R}$.

Keywords: Daugavet property; Centralizer

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1. Introduction

A Banach space X is said to have the Daugavet property if the equality $\|\mathrm{Id}+T\| = 1 + \|T\|$ holds for every bounded linear operator T on X with one-dimensional range. Classical examples of Banach spaces fulfilling the Daugavet property are $\mathcal{C}(K)$, for every perfect compact Hausdorff topological space K, and $L_1(\mu)$, for every non-atomic measure μ. Since Daugavet's pioneering paper [11], the study of Banach spaces having the Daugavet property has attracted the attention of many authors, and today such a study has attained a flourishing development (see [1–4,15,16, 19–23]).

Concerning stability of the Daugavet property, it is known that this property is preserved by taking arbitrary ℓ_1- or c_0-sums [23], and by passing from dual Banach spaces to their preduals. However, unfortunately, not much more is known in this direction. A solution to this handicap consists of the introduction of properties strictly stronger than the one of Daugavet, and which behave better concerning stability (see [8–10,17]).

In the present paper, we follow the line just reviewed. In Section 2, we introduce the notion of a "representable" Banach space (Definition 2.3), prove that representable Banach spaces have the Daugavet property (Lemma 2.4) and that representability passes from a Banach space Y to the complete injective tensor product with an arbitrary Banach space (Corollary 2.6), and to the space of all bounded linear operators from an arbitrary Banach space to Y (Lemma 5).

Sections 3 and 4 of the paper are devoted to show that the class of representable Banach spaces is reasonably wide. We prove that, if K is a perfect compact Hausdorff topological space, if Y is an arbitrary Banach space, if Z is a norming subspace of Y^* for Y, and if τ is a vector space topology on Y with $\sigma(Y,Z) \leqslant \tau \leqslant n$ (where n stands for the norm topology), then the Banach space $\mathcal{C}(K,(Y,\tau))$ is representable (Theorem 3.1). We also prove that, if K is a perfect compact Hausdorff topological space, if Y is an arbitrary Banach space, and if X is a $\mathcal{C}(K,Y)$-superspace (in the sense of [17]) which is also a $\mathcal{C}(K)$-module in the natural way, then X is representable (Theorem 3.4). By the way, Theorems 3.1 and 3.4 are independent (a courtesy of V. Kadets, see Remark 3.3). Finally, we show that every dual Banach space without minimal M-summands is representable (Theorem 4.3). As a consequence, if X and Y are Banach spaces, and if X has no minimal L-summand, then $X \widehat{\otimes}_\pi Y$ has the Daugavet property (Corollary 4.7).

To conclude this introduction, let us point out that the definition of a representable Banach space is quite technical, and is inspired by the theory of the "centralizer" and "function module representations" of Banach spaces [7]. Actually, the proof of Theorem 4.3 relies strongly on this theory.

Notation. Throughout this paper $\mathbb{K}$ will mean the field of real or complex numbers. When the field $\mathbb{K}$ has been fixed, and a compact Hausdorff topological space K has been given, we denote by $\mathcal{C}(K)$ the Banach space of all continuous functions from K to $\mathbb{K}$. Now, let X be a Banach space over $\mathbb{K}$. We denote by B_X, S_X, and X^* the closed unit ball, the unit sphere, and the (topological) dual, respectively, of X. We denote by w the weak topology of X, and by w^* the weak* topology of X in the case that X is a dual Banach space. Now, let Y be another Banach space over $\mathbb{K}$. Then the symbol $\mathcal{L}(X,Y)$ (with the usual convention $\mathcal{L}(X) := \mathcal{L}(X,X)$) will stand for the Banach space of all bounded linear operators from X to Y.