Available online at www.sciencedirect.com
ScienceDirect
Journal of Functional Analysis 257 (2009) 3239–3292
JOURNAL OF Functional Analysis
www.elsevier.com/locate/jfa

K-theory for the maximal Roe algebra of certain expanders

Hervé Oyono-Oyono [a,b], Guoliang Yu [c,*]

[a] *Université Blaise Pascal & CNRS, Clermont-Ferrand, France*
[b] *PIMS, University of Victoria, Canada*
[c] *Vanderbilt University, Nashville, USA*

Received 11 February 2009; accepted 29 April 2009

Available online 26 May 2009

Communicated by Alain Connes

Abstract

We study in this paper the maximal version of the coarse Baum–Connes assembly map for families of expanding graphs arising from residually finite groups. Unlike for the usual Roe algebra, we show that this assembly map is closely related to the (maximal) Baum–Connes assembly map for the group and is an isomorphism for a class of expanders. We also introduce a quantitative Baum–Connes assembly map and discuss its connections to K-theory of (maximal) Roe algebras.

Keywords: Baum–Connes conjecture; Coarse geometry; Expanders; Novikov conjecture; Operator algebra K-theory; Roe algebras

Contents

* Corresponding author.
E-mail addresses: oyono@math.cnrs.fr (H. Oyono-Oyono), guoliang.yu@vanderbilt.edu (G. Yu).

0022-1236/$ – see front matter
doi:10.1016/j.jfa.2009.04.017

1. Introduction

In this paper, we study K-theory of (maximal) Roe algebras for a class of expanders. The Roe algebra was introduced by John Roe in his study of higher index theory of elliptic operators on noncompact spaces [13]. The K-theory of Roe algebra is the receptacle for the higher indices of elliptic operators. If a space is coarsely embeddable into Hilbert space, then K-theory of Roe algebra and higher indices of elliptic operators are computable [18]. Gromov discovered that expanders do not admit coarse embedding into Hilbert space [5]. The purpose of this paper is to completely or partially compute K-theory of the (maximal) Roe algebras associated to certain expanders. In particular, we prove the maximal version of the coarse Baum–Connes conjecture for a special class of expanders. The coarse Baum–Connes conjecture is a geometric analogue of the Baum–Connes conjecture [1] and provides an algorithm of computing K-theory of Roe algebras and higher indices of elliptic operators. We also prove the (maximal) coarse Novikov conjecture for a class of expanders. The coarse Novikov conjecture gives a partial computation of K-theory of Roe algebras and an algorithm to determine non-vanishing of higher indices for elliptic operators. Our results on the coarse Novikov conjecture are more general than results obtained in [3,4,6]. The question of computing K-theory of (maximal) Roe algebras associated to general expanders remains open. We show that this question is closely related to certain quantitative Novikov conjecture and the quantitative Baum–Connes conjecture for the K-theory of (maximal) Roe algebras. We explore this connection to prove the quantitative Novikov conjecture and the quantitative Baum–Connes conjecture in some cases.

The class of expanders under examination in this paper is those associated to a finitely generated and residually finite group Γ with respect to a family

$$\Gamma_0 \supset \Gamma_1 \supset \cdots \supset \Gamma_n \supset \cdots$$

of finite index normal subgroups. The behavior of the Baum–Connes assembly map for Γ and of the coarse Baum–Connes assembly map for the metric space $X(\Gamma) = \coprod_{i \in \mathbb{N}} \Gamma/\Gamma_i$ can differ