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On a stochastic nonlinear equation arising from 1D integro-differential scalar conservation laws

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We dedicate this paper to Jane Truman

Abstract

In this paper we study the initial problem for a stochastic nonlinear equation arising from 1D integro-differential scalar conservation laws. The equation is driven by Lévy space–time white noise in the following form:

$$(\partial_t - A)u + \partial_x q(u) = f(u) + g(u)F_{t,x}$$

for $u : (t, x) \in (0, \infty) \times \mathbb{R} \mapsto u(t, x) \in \mathbb{R}$, where A is an integro-differential operator associated with a symmetric, nonlocal, regular Dirichlet form, and $F_{t,x}$ stands for a Lévy space–time white noise. The problem is interpreted as a stochastic integral equation of jump type involving certain convolution kernels. Existence of a unique local (in time) $L^2(\mathbb{R})$-valued solution is obtained.

Keywords: Stochastic nonlinear equations; Symmetric integro-differential operators; Lévy space–time white noise; Stochastic integral equations of jump type; Local existence and uniqueness

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1. Introduction

This paper is concerned with the initial problem for the following stochastic nonlinear equation

$$(\partial_t - A)u(t,x) + \partial_x\big(q\big(t,x,u(t,x)\big)\big) = f\big(t,x,u(t,x)\big) + g\big(t,x,u(t,x)\big)F_{t,x}$$

on the given domain $[0,\infty)\times\mathbb{R}$ with $L^2(\mathbb{R})$ initial condition, where A is an integro-differential operator, the $L^2(\mathbb{R})$-generator of a symmetric, nonlocal, regular Dirichlet form which generates a strong Feller semigroup $\{e^{sA}\}_{s>0}$ with transition density kernels (cf., e.g., [27,28] and more recently, e.g., [3,13,24] and references therein, and see Section 2.1 for its brief introduction). Here $q:[0,\infty)\times\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ is measurable and corresponds to the "nonlinearity," $f, g:[0,\infty)\times\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ are measurable, and $F_{t,x}$ is the so-called Lévy space–time white noise consisting of Gaussian space–time white noise (i.e., a Brownian sheet on $[0,\infty)\times\mathbb{R}$) and Poisson space–time white noise (see Section 2.2 for the definition). An introductory account of integro-differential scalar conservation laws can be found in [2,30]. Recently, there has been increasing interest in studying integro-differential conservation laws of nonlocal type involving generators of Lévy type (see, e.g., [6–12] and references therein) as well as in studying white noise perturbations of Burgers type nonlinear PDEs with random initial data, see, e.g., [4,5,19,33,37,38] and references therein, or white noise driven stochastic Burgers and fractal Burgers equations, respectively, in [34,35], where the initial problem for both the stochastic Burgers equation and stochastic fractal Burgers equation with Lévy space–time white noise is examined in the mild formulation.

In this paper we introduce a class of stochastic nonlinear equations in one space dimension driven by Lévy space–time white noise which links nonlocal conservation laws involving symmetric integro-differential operators and white noise perturbation of Burgers type nonlinear PDEs considered in the literature mentioned above. We will prove existence of a unique, local, mild solution to the initial problem for the stochastic nonlinear equations we posed. Namely, for any initial function from $L^2(\mathbb{R})$, we obtain a local solution with càdlàg (i.e., right-continuous with left-hand limits in the time variable $t\in[0,\infty)$) trajectories in $L^2(\mathbb{R})$ (see Theorem 3.1).

The paper is organized as follows. In the next section, we elucidate briefly the symmetric integro-differential operators and describe in detail our Lévy white noise. In Section 3, in order to make the problem we are considering precise, we interpret the initial problem for the stochastic nonlinear equation driven by Lévy space–time white noise (weakly) as a jump type stochastic integral equation involving the transition density kernels associated with the symmetric integro-differential operators as the convolution kernels. We present existence of a unique local L^2-solution. Section 4, the final section, is devoted to the proof of Theorem 3.1. Our approach is based on combining the method for solving stochastic Burgers type nonlinear equations driven by Lévy space–time white noise in [34,35] with the techniques for studying nonlocal conservation laws involving Lévy generators developed in [6–12].

2. Preliminaries

2.1. Symmetric integro-differential operators

According to, e.g., [22,25,31,32], a Lévy type operator A is a second-order elliptic pseudo-differential operator having the following representation: