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ScienceDirect
Journal of Functional Analysis 255 (2008) 200–227
JOURNAL OF Functional Analysis
www.elsevier.com/locate/jfa

On non-equilibrium stochastic dynamics for interacting particle systems in continuum

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Received 26 November 2007; accepted 11 December 2007

Available online 1 February 2008

Communicated by Paul Malliavin

Abstract

We propose a general scheme for construction of Markov stochastic dynamics on configuration spaces in continuum. An application to the Glauber-type dynamics with competitions is considered.

Keywords: Configuration space; Glauber dynamics; Non-equilibrium Markov process

1. Introduction

Interacting particle systems (IPS) is a large and growing area of probability theory and infinite-dimensional analysis which is devoted to the study of certain models that arise in statistical physics, biology, economics, etc. Most of the results in the theory of IPS are related to the study of the so-called lattice systems and their Markov stochastic evolutions. In such systems the spatial structure of the considered model is presented by a lattice (or an infinite graph). Considered processes are usually specified by transition rates and associated Markov generators. The existence problem for the corresponding Markov process on the lattice configuration space can be solved positively under quite general assumptions about the transition rates, see e.g. [22].

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0022-1236/$ – see front matter
doi:10.1016/j.jfa.2007.12.006

Comparing with the lattice case, the situation with Markov stochastic dynamics for IPS in continuum is essentially different. In particular, it is true for an important class of birth-and-death processes in continuum (or so-called spatial birth-and-death processes). To this class belongs the Glauber type dynamics in continuum which are under active consideration, see [3,29]. Another class of interesting stochastic processes is formed by the Kawasaki type dynamics in continuum [19] and gradient diffusions [1,17]. Most of the results we have up to now for these processes are related to the equilibrium case (via the Dirichlet forms approach) [13,18] or to the processes in bounded domains (see, e.g., [8,24]). The situation with the non-equilibrium case is much pure. In particular, non-equilibrium spatial birth-and-death processes were constructed recently by Garcia and Kurtz for a special class of transition rates using techniques of stochastic differential equations [6] and a graphical construction was applied in [5]. Note that in both mentioned papers the death rate was considered to be a constant and the latter plays an essential technical role. A continuous version of the lattice contact model was analyzed in [14].

In contrast to the lattice case, constructions of the stochastic dynamics in continuum show essential difference between the Markov processes and Markov functions concepts. The latter notion (due to E. Dynkin) concerns the case of the processes with given initial distributions contrary to the more usual initial points framework. This weaker notion of the Markov function is not so essential in the lattice models because corresponding Markov processes can be constructed (typically) under very general assumptions. A principal role of dynamics with given classes of initial distributions was clarified at first for (deterministic) Hamiltonian dynamics in continuum, see e.g. [4].

In the present paper the role of the Markov functions approach is clarified for an infinite particle stochastic dynamics in continuum. This approach is based on the study of the corresponding (dual) Kolmogorov equation on measures. Such equation can be transported to an equation for corresponding correlation functions. Typically, this correlation functions equation does not admit a direct perturbation theory approach. In fact, the main technical observation made in the paper is related to the consideration of its dual time evolution on the so-called quasi-observables. This approach appeared for the first time in the literature on stochastic IPS in continuum in our paper [16] in the particular case of a Glauber-type dynamics. The idea to move the dynamics to a proper quasi-observables space (as well as the notion of quasi-observables itself) follows naturally from the concepts of harmonic analysis on configuration spaces, see e.g. [11]. We apply a perturbation technique to this dynamics in proper weighted L^1-spaces of functions on finite configurations and produce time evolutions of correlation functions as a dual object. The choice of corresponding weights gives precise description of the class of admissible initial distributions for our processes.

One should emphasize also another principal moment of the paper. Namely, even if we have constructed a time evolution of the correlation functions, we need to show that they correspond to a time evolution of measures. In fact, this point is hidden in several works in statistical physics concerning BBGKY-hierarchy, etc. A rigorous mathematical analysis of this problem is based on a proper concept of positive definiteness of the correlation functions which was developed in [2,11].

The power of the described general scheme we illustrate by the application to a particular model of Glauber-type stochastic dynamics in continuum. In this process the birth of points is independent and uniformly distributed in space. Without a death part, the density of the system will grow to the infinity with the time. To prevent such unbounded growth we can introduce a self-regulation in the model. The latter can be done in several ways and one of them is to introduce the competition between points via a proper death rate. This competition (via density