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Representation theory of Drinfeld modular forms of level T

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ABSTRACT

This paper expands upon our results for the arithmetic of Drinfeld modular forms of level T [12] by providing an interpretation from a representation theoretic point of view.
We identify $\mathrm{GL}(2,\mathbb{F}_q)$-modules that arise naturally from the theory of Drinfeld modular forms of level T with classical $\mathrm{GL}(2,\mathbb{F}_q)$-modules. Using the arithmetic of the so-called modified Eisenstein series all isomorphisms are stated explicitly. In particular, we examine the close connection between Drinfeld modular forms of level T and the theory of symmetric powers of the tautological representation of $\mathrm{GL}(2,\mathbb{F}_q)$ described in previous work [13].

0. Introduction

Drinfeld modular forms are the function field analogue to classical elliptic modular forms. The origins of this theory go back to Drinfeld's central work [5], which has been made more accessible to a wider audience by Deligne and Husemöller [4]. For early work on function field modular forms see for example Goss [9,10] or Gekeler [6].

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However, until now Drinfeld modular forms have not been studied from a representation theoretic point of view. In the present paper we examine the interaction of these two concepts.

Group actions arise naturally from the definition of Drinfeld modular forms. We focus on the case of modular forms of level T (where $A = \mathbb{F}_q[T]$ is the base ring) for two reasons: On the one hand, the structure of the algebra of modular forms of level T is well-known. The principal results are due to Cornelissen, see [3]. I have expanded upon these in [12]. On the other hand, restriction to level T allows us to focus on actions of the group $G = \mathrm{GL}(2, \mathbb{F}_q)$; the modular representation of which has been studied extensively before, see for example [2] and [14].

The basic idea of our approach can be applied to the general case of modular forms of level N. However, both the algebra of modular forms as well as the group action will be more complicated in this case. Therefore, the generalization of the present results is non-trivial.

The aim of the present paper is to determine the G-module structure of naturally occurring modules of Drinfeld modular forms. Specifically, we provide identifications with classical G-modules such as symmetric powers of the tautological two-dimensional module V. In general our results are given explicitly in terms of distinguished bases.

The principal results are Theorem 2.5, which describes the modules of Eisenstein series, Theorem 3.2, in which we state the connection between modular forms of weight k and certain symmetric powers of V, Theorem 4.7 and Theorem 4.14, which describe the smallest non-trivial submodule and the successive quotients of the cusp filtration, respectively, and Theorem 5.3, in which we identify modules of cusp forms with twisted symmetric powers of V.

The present paper is a summary of chapters 7 through 9 of the author's dissertation [11], with some additional content from chapter 10.

Also, it is a continuation of prior articles [12] and [13]. These preliminaries are briefly summarized in section 1 of the present work. Specifically, we fix notation for the objects of interest in the Drinfeld setting and for the necessary tools from modular representation theory.

In the second section we study the action of G on the module of Eisenstein series, which is central to the results of the following sections.

In the third section we show that the module of modular forms of weight k is isomorphic to $\mathrm{Sym}^{kq}(V)$.

The fourth section deals with the cusp filtration. We study relations between filtrations of different weights and describe the G-module structure of the successive quotients using results from [12].

In the final section we apply results from [13] to identify modules of cusp forms with determinant twists of symmetric powers of V.