Journal of Number Theory 168 (2016) 374–385

Contents lists available at ScienceDirect

Journal of Number Theory

www.elsevier.com/locate/jnt

Strictly regular ternary Hermitian forms

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ARTICLE INFO

Article history:
Received 28 October 2015
Received in revised form 8 April 2016
Accepted 9 April 2016
Communicated by David Goss

MSC:
primary 11E39
secondary 11E12, 11E20

Keywords:
Ternary Hermitian forms
Strictly regular Hermitian forms

ABSTRACT

A positive definite integral Hermitian form is called strictly regular if it primitively represents all integers that can be primitively represented locally everywhere by the form itself. In this article, we show that there are only finitely many equivalence classes of primitive strictly regular positive definite integral ternary Hermitian forms over a fixed imaginary quadratic field.

1. Introduction

A main question in the study of positive definite integral quadratic forms is determining which integers are represented by a given form. Specifically, given a positive definite integral quadratic form, Q, for which positive integers a are there integers $x_1, \ldots, x_n$ such that $Q(x_1, \ldots, x_n) = a$? In order for such a representation to exist it is necessary that there are local representations over the p-adic completion $\mathbb{Z}_p$ of $\mathbb{Z}$ for every prime p;

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http://dx.doi.org/10.1016/j.jnt.2016.04.012

however, this is not sufficient. When we do have sufficiency we call the forms *regular*. Regular quadratic forms were first studied by Dickson in [2].

It is known that primitive regular positive definite integral ternary quadratic forms lie in only finitely many equivalence classes, but the same is not true for primitive regular positive definite integral quaternary quadratic forms [4]. In the quaternary case, Earnest, Kim and Meyer [6] are able to achieve finiteness by restricting their attention to primitive strictly regular positive definite integral quaternary quadratic forms. A positive definite integral quadratic form Q is said to be *strictly regular* if for every positive integer a there exists a primitive representation by Q over $\mathbb{Z}$ whenever there is a primitive representation by Q over $\mathbb{Z}_p$ for every prime p.

There are similar finiteness results for regular positive definite integral Hermitian forms. Earnest and Khosravani [5] show that there are only finitely many classes of primitive regular positive definite integral binary Hermitian forms over a fixed imaginary quadratic field. More precisely, they deal with the more general notion of Hermitian lattices and show that for a positive definite integral binary Hermitian lattice L, the cardinality of $\mathcal{E}(L)$ tends to infinity as the volume of L tends to infinity, where $\mathcal{E}(L)$ is the set of integers represented by L locally everywhere but not represented by L itself. Chan and Rokicki [1] show that for a fixed totally real number field F of odd degree over $\mathbb{Q}$, there are only finitely many CM extensions E/F for which there exists a regular normal positive definite integral Hermitian lattice over the ring of integers of E. In particular, they have shown that a regular normal positive definite integral binary Hermitian lattice exists over the field $\mathbb{Q}(\sqrt{-m})$ if and only if m is

$$1, 2, 3, 5, 6, 7, 10, 11, 15, 19, 23 \text{ or } 31.$$

Similarly, Kim, Kim and Park [11] find that a primitive regular subnormal positive definite integral binary Hermitian lattice with $\mathfrak{n}L = 2\mathcal{O}$ exists over the field $\mathbb{Q}(\sqrt{-m})$ if and only if m is

$$1, 2, 5, 6, 10, 13, 14, 17, 21, 22, 29, 34, 37 \text{ or } 38.$$

Furthermore, in [12], they show that primitive regular subnormal positive definite integral binary Hermitian lattices with $\mathfrak{n}L = m\mathcal{O}$ appear over infinitely many imaginary quadratic fields $\mathbb{Q}(\sqrt{-m})$.

Our first result concerns the finiteness of primitive regular positive definite integral ternary Hermitian lattices.

Theorem 1.1. *An imaginary quadratic field supports a primitive regular positive definite integral binary Hermitian lattice if and only if it supports infinitely many isometry classes of primitive regular positive definite integral ternary Hermitian lattices.*

We will then concentrate our attention on strict regularity, and the main objective of this paper is to prove the following: