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Determination of elliptic curves by their adjoint p-adic L-functions

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ABSTRACT

Fix p an odd prime. Let E be an elliptic curve over $\mathbb{Q}$ with semistable reduction at p. We show that the adjoint p-adic L-function of E evaluated at infinitely many integers prime to p completely determines up to a quadratic twist the isogeny class of E. To do this, we prove a result on the determination of isobaric representations of $\mathrm{GL}(3, \mathbb{A}_{\mathbb{Q}})$ by certain L-values of p-power twists.

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1. Introduction

In this paper we will prove the following result concerning the p-adic L-function of the symmetric square of an elliptic curve over $\mathbb{Q}$, denoted $L_p(Sym^2\, E, s)$ for $s \in \mathbb{Z}_p$. More specifically, Theorem 1 gives a generalization of the result obtained in [14] concerning p-adic L-functions of elliptic curves over $\mathbb{Q}$:

Theorem 1. *Let p be an odd prime and E, E' be elliptic curves over $\mathbb{Q}$ with semistable reduction at p. Suppose*

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$$L_p(Sym^2 E, n) = CL_p(Sym^2 E', n) \tag{1.1}$$

for all integers n prime to p in an infinite set Y and some constant $C \in \overline{\mathbb{Q}}$. Then E' is isogenous to a quadratic twist E_D of E. If E and E' have square free conductors, then E and E' are isogenous over $\mathbb{Q}$.

Suppose E has good reduction at p. We follow the definition in [5] of the p-adic L-function for the symmetric square of an elliptic curve E over $\mathbb{Q}$ which is defined as the Mazur–Mellin transform of a p-adic measure $\mu_p := \mu_p(E)$. If $\chi : \mathbb{Z}_p^\times \to \mathbb{C}_p^\times$ is a non-trivial wild p-adic character of conductor p^{m_χ}, which can be identified with a primitive Dirichlet character, then

$$L_p(Sym^2 E, \chi) = \int\limits_{\mathbb{Z}_p^\times} \chi \mathrm{d}\mu_p = C_E \cdot \alpha_p^{-2m_\chi} \tau(\overline{\chi})^2 p^{m_\chi} L(Sym^2 E, \chi, 2) \tag{1.2}$$

where C_E is a constant that depends on E, $\tau(\chi)$ is the Gauss sum of χ and α_p is a root of the polynomial $X^2 - a_p X + p$, with $a_p = p + 1 - \#E(\mathbb{F}_p)$. It is proved in [5] that if E has good ordinary reduction at p then $\mu_p(E)$ is a bounded measure on $\mathbb{Z}_p^\times$, while if E has good supersingular reduction at p then $\mu_p(E)$ is h-admissible with $h = 2$ (cf. [23]).

Similarly, if E has bad multiplicative reduction at p, then for a non-trivial even Dirichlet character as above we have

$$L_p(Sym^2 E, \chi) = \int\limits_{\mathbb{Z}_p^\times} \chi \mathrm{d}\mu_p = C'_E \tau(\overline{\chi})^2 p^{m_\chi} L(Sym^2 E, \chi, 2), \tag{1.3}$$

with $\mu_p(E)$ a bounded measure on $\mathbb{Z}_p^\times$.

Set

$$L_p(Sym^2 E, \chi, s) := L_p(Sym^2 E, \chi \cdot \langle x \rangle^s)$$

where $\langle \cdot \rangle : \mathbb{Z}_p^\times \to 1 + p\mathbb{Z}_p$, with $\langle x \rangle = \frac{x}{\omega(x)}$ and $\omega : \mathbb{Z}_p^\times \to \mathbb{Z}_p^\times$ the Teichmüller character.

Using the theory of h-admissible measures developed in [23], by Lemma 4 in Section 6 identity (1.1) implies that

$$L_p(Sym^2 E, \chi, s) = CL_p(Sym^2 E', \chi, s)$$

holds for all $s \in \mathbb{Z}_p$ and χ a wild p-adic character.

Let f, f' be the newforms of weight 2 associated to E and E', and π, π' the unitary cuspidal automorphic representations of $\mathrm{GL}(2, \mathbb{A}_\mathbb{Q})$ generated by f and f' respectively. Then

$$L(Sym^2 E, s) = L(Sym^2 \pi, s - 1) \tag{1.4}$$

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