Journal of Number Theory 149 (2015) 15–32

Contents lists available at ScienceDirect

Journal of Number Theory

www.elsevier.com/locate/jnt

The third order variations on the Fibonacci universal code

Ayse Nalli *, Cagla Ozyilmaz

Department of Mathematics, Karabuk University, Balıklarkayası Mevkii, 78050, Turkey

ARTICLE INFO

Article history:
Received 13 January 2014
Received in revised form 15 July 2014
Accepted 15 July 2014
Available online 28 September 2014
Communicated by David Goss

MSC:
11B37
11B39

Keywords:
Cryptography
Gopala–Hemachandra code
Fibonacci representation

ABSTRACT

Text. In this paper, we have studied the third order variations on the Fibonacci universal code and we have displayed tables $GH_a^{(3)}(n)$ we have defined for $-20 \leq a \leq -2$ and $1 \leq n \leq 100$. Also, we have compared with the third order variations on the Fibonacci universal code and the second order variations on the Fibonacci universal code [2] in terms of cryptography and we have found that the third order variations on the Fibonacci universal code are more advantageous than the second order variations on the Fibonacci universal code.

Video. For a video summary of this paper, please visit http://youtu.be/i2A4LZl9nNE.

1. Introduction

Fibonacci coding is based on Fibonacci numbers and was defined by Apostolico and Fraenkel (1987) [1]. Fibonacci numbers of order $m \geq 2$, denoted by $F_i^{(m)}$, are defined by following recurrence relation:

* Corresponding author.
E-mail addresses: aysenalli@karabuk.edu.tr (A. Nalli), cagla.zylmz@gmail.com (C. Ozyilmaz).

http://dx.doi.org/10.1016/j.jnt.2014.07.010

Table 1
The second and the third order Standard Fibonacci representation for $1 \leq n \leq 15$.

n	2 Zechendorf (for $m = 2$ Standard Fibonacci representation)	3 Zechendorf (for $m = 3$ Standard Fibonacci representation)
1	1	1
2	01	01
3	001	11
4	101	001
5	0001	101
6	1001	011
7	0101	0001
8	00001	1001
9	10001	0101
10	01001	1101
11	00101	0011
12	10101	1011
13	000001	00001
14	100001	10001
15	010001	01001

$$F_n^{(m)} = F_{n-1}^{(m)} + F_{n-2}^{(m)} + \cdots + F_{n-m}^{(m)}$$

for $n > 0$ and the boundary conditions $F_0^{(m)} = 1$ and $F_n^{(m)} = 0$ for $-m < n < 0$ [7]. Fibonacci coding is a universal code which encodes positive integers into binary codewords. Every positive integer has a unique representation as the sum of nonconsecutive Fibonacci numbers according to Zeckendorf's theorem [10]. Let us first consider the Standard Fibonacci numbers of order $m = 2$.

Any positive integer B can be represented by a binary string of length r, $c_1 \cdots c_r$, such that $B = \sum_{i=1}^{r} c_i F_i^{(2)}$. The representation will be unique if one uses the following procedure to produce it: given the integer B, find the largest Fibonacci number $F_r^{(2)}$ smaller or equal to B; then continue recursively with $B - F_r^{(2)}$. For example $16 = 3 + 13$, so its binary Fibonacci representation would be 001001.

As a result of this encoding procedure, there are never consecutive Fibonacci numbers in any of these sums, implying that in the corresponding binary representation, there are no adjacent 1 bits.

The generalization to higher order seems at first sight straightforward: any integer B can be uniquely represented by the string $d_1 \cdots d_s$ such that $B = \sum_{i=1}^{s} d_i F_i^{(m)}$ using the iterative encoding procedure mentioned above. In this representation, there are no consecutive substrings of m 1 bits [7]. In this paper, we, in particular, have studied Fibonacci numbers of order $m = 3$.

We append $(m - 1)$ 1 bits to the Fibonacci representation of n so as to construct the Fibonacci code of n whose order is m. (See Tables 1 and 2.)