



Contents lists available at ScienceDirect

Journal of Number Theory

www.elsevier.com/locate/jnt



Distinction of depth-zero representations



Fiona Murnaghan¹

*Department of Mathematics, University of Toronto, 40 Saint George Street,
Toronto, M5S 2E4, Canada*

ARTICLE INFO

Article history:

Received 1 July 2013

Received in revised form 10

February 2014

Accepted 5 March 2014

Available online 14 March 2014

Communicated by James Cogdell,

Hervé Jacquet, Dihua Jiang, and

Steve Kudla

Keywords:

Distinguished representation

Involution

Reductive p -adic group

Depth-zero representation

K -type

ABSTRACT

Let π be a depth-zero irreducible admissible representation of a connected reductive p -adic group G . Let H be the group of fixed points of an involution θ of G . We relate H -distinction of π to existence of minimal K -types of π that exhibit particular symmetry properties relative to θ . In addition, we show that when π is H -distinguished, then (up to conjugacy) the support of π is of the form (M, τ) where M is a θ -stable Levi subgroup of G and τ is a depth-zero irreducible supercuspidal representation of M . Moreover, τ contains a minimal K -type (M_x, ρ) such that M_x is a θ -stable maximal parahoric subgroup of M and ρ is the inflation of a distinguished cuspidal representation of the quotient of M_x by its pro-unipotent radical.

© 2014 Elsevier Inc. All rights reserved.

1. Introduction

If π is a representation of a group G , H is a subgroup of G and χ is a one-dimensional representation of H , let $\text{Hom}_H(\pi, \chi)$ be the space of linear functionals λ on the space V of π such that $\lambda(\pi(h)v) = \chi(h)\lambda(v)$ for all $h \in H$ and $v \in V$. When $\text{Hom}_H(\pi, \chi)$ is nonzero, we say that π is (H, χ) -distinguished, or simply H -distinguished if χ is trivial. If the square of χ is trivial, then we say that π is quadratically distinguished.

E-mail address: fiona@math.toronto.edu.

¹ The author's research is supported by NSERC Discovery Grant RGPIN 155502.

Let F be a local nonarchimedean field of odd residual characteristic and let \mathbf{G} be a connected reductive F -group. Let θ be an involution of $G = \mathbf{G}(F)$, that is, an F -automorphism of \mathbf{G} of order two, let $\mathbf{H} = \mathbf{G}^\theta$ and $H = \mathbf{H}(F)$. The H -distinguished irreducible admissible representations of G , along with their spherical characters (see [17]), are among the basic objects in harmonic analysis on the p -adic symmetric space G/H . In addition, when F is the completion of a number field over which \mathbf{G} and θ are defined, H -distinguished representations of G often occur as the local components of globally distinguished automorphic representations and are related to aspects of the Langlands program.

In this paper, we study (H, χ) -distinction of smooth representations of G which are generated by their isotypic subspaces associated to unrefined minimal K -types of depth zero. Let p be the residual characteristic of F . Following Spice [18], we say that an element g of G is topologically p -unipotent if $\lim_{n \rightarrow \infty} g^{p^n} = 1$. We assume that χ is a quasicharacter of H that is trivial on the set of topologically p -unipotent elements in H . In particular, the depth of χ (in the sense of [13]) is zero. We find that such (H, χ) -distinguished representations contain unrefined minimal K -types of the form (K, ρ) where $K \cap \theta(K)$ is large relative to K (see below) and ρ is $(K \cap H, \chi |_{K \cap H})$ -distinguished. Before giving a more detailed description of the results, we discuss relations between distinction of representations and distinction of K -types contained in those representations.

We say that an irreducible supercuspidal representation π of G is tame if π has depth zero or if \mathbf{G} splits over a tamely ramified extension of F and π is one of the supercuspidal representations constructed by Yu [19]. Under some assumptions regarding quasicharacters of twisted Levi subgroups of G , in [4], J. Hakim and the author derived necessary conditions for H -distinction of a tame supercuspidal representation π of G . These conditions can be expressed in terms of data defined in [19] which are used in the construction of π . This is connected to the behavior under θ of inducing data for π . In particular, a tame supercuspidal representation π of G is H -distinguished if and only if there exist a θ -stable open compact-mod-center subgroup K of G and a representation ρ of K such that π is equivalent to the representation $\text{c-ind}_K^G \rho$ obtained via compact induction from ρ and $\text{Hom}_{K^\theta}(\rho, 1)$ is nonzero, where $K^\theta = K \cap G^\theta$. In addition, K^θ -distinction of ρ reduces to quadratic distinction of a representation that arises as the inducing data for a twist of particular depth-zero supercuspidal representation of a θ -stable reductive subgroup (a tamely ramified twisted Levi subgroup) of G .

In general, K is noncompact, so strictly speaking (K, ρ) is not a K -type. However the restriction of ρ to a compact open subgroup of K is a K -type. It can be useful to have specific information about properties, such as quadratic distinction, of particular K -types contained in distinguished representations. For example, in [14], the results of [4] were applied to deduce results about existence of distinguished tame supercuspidal representations.

Recall that if π is an irreducible smooth representation of G then π occurs as a composition factor of a representation $\text{Ind}_P^G \tau$, where τ is an irreducible supercuspidal

Download English Version:

<https://daneshyari.com/en/article/4593719>

Download Persian Version:

<https://daneshyari.com/article/4593719>

[Daneshyari.com](https://daneshyari.com)