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On local descent for unitary groups

David Soudry [*,1], Yaacov Tanay

School of Mathematical Sciences, Sackler Faculty of Exact Sciences, Tel-Aviv University, 69978 Israel

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A B S T R A C T

We study the local descent from irreducible, supercuspidal, self-conjugate representations of $\mathrm{GL}_{2n}(E)$ to irreducible, supercuspidal and generic representations of the quasi-split unitary group $\mathrm{U}_{2n}(F)$ in $2n$ variables, corresponding to a quadratic extension E/F of p-adic fields. We construct the descent and prove that it is nontrivial, supercuspidal, generic and irreducible. We write the relations with poles of local gamma factors and functoriality. We also consider representations as above of $\mathrm{GL}_{2n+1}(E)$.

* Corresponding author.
E-mail addresses: soudry@post.tau.ac.il (D. Soudry), jacques@post.tau.ac.il (Y. Tanay).

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1. Introduction

Let F be a p-adic field and let E/F be a quadratic extension. Consider $\mathrm{U}_{2n}(F)$, the quasi-split unitary group in $2n$ variables attached to E/F. In this paper, we introduce and study the local descent from irreducible, supercuspidal, self-conjugate representations of $\mathrm{GL}_{2n}(E)$ to irreducible, supercuspidal, generic representations of $\mathrm{U}_{2n}(F)$. In little more details, we start with an irreducible, supercuspidal representation τ of $\mathrm{GL}_{2n}(E)$, such that its local Asai L-function, $L(\tau, Asai, s)$, has a pole at $s = 0$. Then we construct an irreducible, supercuspidal and generic representation σ of $\mathrm{U}_{2n}(F)$, such that $\gamma(\sigma \times (\tau \otimes \lambda), s, \psi)$ has a pole at $s = 1$, the last function being the local gamma factor for $\mathrm{U}_{2n}(F) \times \mathrm{GL}_{2n}(E)$, arising from global Rankin–Selberg integrals, which represent the standard L-function. In the local gamma factor, τ is twisted by a character λ of E^*, whose restriction to F^* is $\omega_{E/F}$, the unique nontrivial (quadratic) character of F, which is trivial on $N_{E/F}(F^*)$. We fix λ once and for all. Also, ψ which appears in gamma is a nontrivial character of F, which we fix throughout the paper. We prove that σ as above is unique, up to isomorphism (for given τ, λ and ψ).

In order to be more specific, let us introduce some notations. Denote by $x \mapsto \bar{x}$ the Galois conjugation of E/F. For a matrix $a \in M_{k\times r}(E)$, we will denote by $\bar{a}$ the matrix in $M_{k\times r}(E)$, whose (i,j) coordinate is $\bar{a}_{i,j}$; $1 \leqslant i \leqslant k$, $1 \leqslant j \leqslant r$. Denote by ψ_E the character of E, defined by

$$\psi_E(x) = \psi\left(\frac{1}{2}\, tr_{E/F}(x)\right), \quad x \in E. \tag{1.1}$$

We will realize $\mathrm{U}_{2n}(F)$ as

$$\mathrm{U}_{2n}(F) = \left\{g \in \mathrm{GL}_{2n}(E) \;\middle|\; gJ_{2n}{}^t\bar{g} = J_{2n}\right\},$$

where

$$J_{2n} = \begin{pmatrix} & w_n \\ -w_n & \end{pmatrix}; \qquad w_n = \begin{pmatrix} & & & 1 \\ & & 1 & \\ & \cdots & & \\ 1 & & & \end{pmatrix}_{n\times n}. \tag{1.2}$$

The coordinates of the matrix w_n are all zero, except those on the main skew diagonal, in which all coordinates are 1. Let V_n be the standard maximal unipotent subgroup of $\mathrm{U}_{2n}(F)$. It consists of upper unipotent matrices. The standard non-degenerate (or Whittaker) character of V_n, corresponding to ψ is the following character of V_n

$$\psi_{V_n}(v) = \psi_E\left(\sum_{i=1}^{n} v_{i,i+1}\right), \quad v \in V_n. \tag{1.3}$$