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Signed Shintani cones for number fields with one complex place ☆

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ABSTRACT

We give a signed fundamental domain for the action on $\mathbb{C}^* \times \mathbb{R}_+^{n-2}$ of the totally positive units $E(k)_+$ of a number field k of degree n and having exactly one pair of complex embeddings. This signed fundamental domain, built of k-rational simplicial cones, is as convenient as a true fundamental domain for the purpose of studying Dedekind zeta functions. However, while there is no general construction of a true fundamental domain, we construct a signed fundamental domain from any set of fundamental units of k.

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1. Introduction

Motivated by the study of special values of L-functions over totally real number fields, Shintani introduced in 1976 [Sh1] a geometric method that allowed him to write any partial zeta function of a totally real number field as a finite sum of certain Dirichlet series, which can be considered as a natural generalization of the Hurwitz zeta function. Later [Sh2] Shintani extended these results to general number fields. In order to enunciate Shintani's geometric method, fix a number field k with r real embeddings and s pairs of complex embeddings (*i.e.* $[k:\mathbb{Q}] = 2s + r$), and let $E(k)$ be its group of units. Given a complete set $\tau_i : k \to \mathbb{C}$ $(1 \le i \le s + r)$ of embeddings of k,

$$\underbrace{\tau_1, \bar{\tau}_1, \tau_2, \bar{\tau}_2, \ldots, \tau_s, \bar{\tau}_s}_{\text{complex embeddings}}, \underbrace{\tau_{s+1}, \tau_{s+2}, \ldots, \tau_{s+r}}_{\text{real embeddings}}, \tag{1}$$

we can consider $k \subset \mathbb{C}^s \times \mathbb{R}^r$ by identifying $x \in k$ with

$$\left(x^{(1)}, x^{(2)}, \ldots, x^{(s+r)}\right) \in \mathbb{C}^s \times \mathbb{R}^r,$$

where $x^{(i)} := \tau_i(x)$. Put

$$E(k)_+ := E(k) \cap \left(\mathbb{C}^s \times \mathbb{R}^r_+\right) \quad \text{and} \quad k_+ := k \cap \left(\left(\mathbb{C}^*\right)^s \times \mathbb{R}^r_+\right),$$

where $\mathbb{R}^r_+ := (0, \infty)^r$. Then the group $E(k)_+$ of totally positive units of k acts on $(\mathbb{C}^*)^s \times \mathbb{R}^r_+$ by component-wise multiplication, where $(\mathbb{C}^*)^s := (\mathbb{C} \smallsetminus \{0\})^s$. On the other hand, if $v_1, v_2, \ldots, v_d \in \mathbb{C}^s \times \mathbb{R}^r$ $(1 \le d \le 2s + r)$ is a set of $\mathbb{R}$-linearly independent vectors, we shall call

$$C(v_1, v_2, \ldots, v_d) := \{t_1 v_1 + t_2 v_2 + \cdots + t_d v_d \mid t_i > 0\}$$

the *d-dimensional simplicial cone generated by* $v_1, v_2, \ldots, v_d$.