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Signed Shintani cones for number fields with one complex place [☆]



Milton Espinoza

Departamento de Matemáticas, Universidad de Valparaíso, Gran Bretaña 1091, Valparaíso, Chile

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ABSTRACT

We give a signed fundamental domain for the action on $\mathbb{C}^* \times \mathbb{R}_+^{n-2}$ of the totally positive units $E(k)_+$ of a number field k of degree n and having exactly one pair of complex embeddings. This signed fundamental domain, built of k -rational simplicial cones, is as convenient as a true fundamental domain for the purpose of studying Dedekind zeta functions. However, while there is no general construction of a true fundamental domain, we construct a signed fundamental domain from any set of fundamental units of k .

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E-mail address: milton.espinoza@yahoo.com.

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1. Introduction

Motivated by the study of special values of L -functions over totally real number fields, Shintani introduced in 1976 [Sh1] a geometric method that allowed him to write any partial zeta function of a totally real number field as a finite sum of certain Dirichlet series, which can be considered as a natural generalization of the Hurwitz zeta function. Later [Sh2] Shintani extended these results to general number fields. In order to enunciate Shintani’s geometric method, fix a number field k with r real embeddings and s pairs of complex embeddings (*i.e.* $[k : \mathbb{Q}] = 2s + r$), and let $E(k)$ be its group of units. Given a complete set $\tau_i : k \rightarrow \mathbb{C}$ ($1 \leq i \leq s + r$) of embeddings of k ,

$$\underbrace{\tau_1, \bar{\tau}_1, \tau_2, \bar{\tau}_2, \dots, \tau_s, \bar{\tau}_s}_{\text{complex embeddings}}, \underbrace{\tau_{s+1}, \tau_{s+2}, \dots, \tau_{s+r}}_{\text{real embeddings}} \tag{1}$$

we can consider $k \subset \mathbb{C}^s \times \mathbb{R}^r$ by identifying $x \in k$ with

$$(x^{(1)}, x^{(2)}, \dots, x^{(s+r)}) \in \mathbb{C}^s \times \mathbb{R}^r,$$

where $x^{(i)} := \tau_i(x)$. Put

$$E(k)_+ := E(k) \cap (\mathbb{C}^s \times \mathbb{R}_+^r) \quad \text{and} \quad k_+ := k \cap ((\mathbb{C}^*)^s \times \mathbb{R}_+^r),$$

where $\mathbb{R}_+^r := (0, \infty)^r$. Then the group $E(k)_+$ of totally positive units of k acts on $(\mathbb{C}^*)^s \times \mathbb{R}_+^r$ by component-wise multiplication, where $(\mathbb{C}^*)^s := (\mathbb{C} \setminus \{0\})^s$. On the other hand, if $v_1, v_2, \dots, v_d \in \mathbb{C}^s \times \mathbb{R}^r$ ($1 \leq d \leq 2s + r$) is a set of \mathbb{R} -linearly independent vectors, we shall call

$$C(v_1, v_2, \dots, v_d) := \{t_1 v_1 + t_2 v_2 + \dots + t_d v_d \mid t_i > 0\}$$

the d -dimensional simplicial cone generated by v_1, v_2, \dots, v_d .

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