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An integral representation, complete monotonicity, and inequalities of Cauchy numbers of the second kind

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In the paper, the author establishes an integral representation, finds the complete monotonicity, minimality, and logarithmic convexity, and presents some inequalities of Cauchy numbers of the second kind.

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1. Introduction

According to [1, pp. 293–294], there are two kinds of Cauchy numbers which may be defined respectively by

$$C_n = \int_0^1 \langle x \rangle_n \,\mathrm{d}x \quad \text{and} \quad c_n = \int_0^1 (x)_n \,\mathrm{d}x, \tag{1.1}$$

where

$$\langle x \rangle_n = \begin{cases} x(x-1)(x-2)\cdots(x-n+1), & n \ge 1 \\ 1, & n = 0 \end{cases} \tag{1.2}$$

and

$$(x)_n = \begin{cases} x(x+1)(x+2)\cdots(x+n-1), & n \ge 1 \\ 1, & n = 0 \end{cases} \tag{1.3}$$

are respectively called the falling and rising factorials. The coefficients expressing rising factorials $(x)_n$ in terms of falling factorials $\langle x \rangle_n$ are called Lah numbers. Lah numbers have an interesting meaning in combinatorics: they count the number of ways a set of n elements can be partitioned into k nonempty linearly ordered subsets. Shortly speaking, Cauchy numbers play important roles in some fields, such as approximate integrals, Laplace summation formula, and difference-differential equations, and are also related to some famous numbers such as Stirling numbers, Bernoulli numbers, and harmonic numbers. Therefore, Cauchy numbers deserve to be studied.

It is known [1, p. 294] that Cauchy numbers of the second kind c_k may be generated by

$$\frac{-t}{(1-t)\ln(1-t)} = \sum_{n=0}^{\infty} c_n \frac{t^n}{n!} \tag{1.4}$$

which is equivalent to

$$\frac{t}{(1+t)\ln(1+t)} = \sum_{n=0}^{\infty} (-1)^n c_n \frac{t^n}{n!}. \tag{1.5}$$

The first few Cauchy numbers of the second kind c_k are

$$c_0 = 1, \qquad c_1 = \frac{1}{2}, \qquad c_2 = \frac{5}{6}, \qquad c_3 = \frac{9}{4}, \qquad c_4 = \frac{251}{30},$$
$$c_5 = \frac{475}{12}, \qquad c_6 = \frac{19087}{84}. \tag{1.6}$$