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A new method to compute the terms of generalized order-k Fibonacci numbers

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ABSTRACT

In this paper, we give a new method to determine the terms of generalized order-k Fibonacci numbers and Fibonacci type sequences by using the inverse of various Hessenberg and triangular matrices. In addition, instead of obtaining the n-th term only, we are able to determine successive $(n+1)$ terms of these sequences with this method simultaneously.

1. Introduction

There is quite an interest in the theory and applications of Fibonacci numbers and their generalizations. Miles [9] defined generalized order-k Fibonacci numbers as

$$f_{k,n} = \sum_{j=1}^{k} f_{k,n-j} \tag{1}$$

for $n > k \geqslant 2$, with boundary conditions: $f_{k,1} = f_{k,2} = f_{k,3} = \cdots = f_{k,k-2} = 0$ and $f_{k,k-1} = f_{k,k} = 1$.

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Since calculating any term of these sequences by recurrence relation is very difficult, there is a need to find other methods. Many researchers obtained terms of Fibonacci numbers and generalized order-k Fibonacci numbers by using determinant of different Hessenberg matrices. For example, Öcal et al. [10] gave some determinantal and permanental representations of k-generalized Fibonacci numbers and obtained Binet's formulas for these sequences. Kılıç and Taşcı [5] studied on permanents and determinants of Hessenberg matrices. Yılmaz and Bozkurt [12] derived permanents and determinants of a type of Hessenberg matrices. Kaygısız and Şahin [2–4] gave some determinantal and permanental representations of Fibonacci type numbers. Li et al. [8] gave three new Fibonacci Hessenberg matrices and obtained determinants of them. In [7,6], authors defined two $(0,1)$-matrices and showed that the permanents of these matrices are the generalized Fibonacci and Lucas numbers.

Chen and Yu [1] presented three Hessenberg matrices

$$H=\begin{bmatrix} h_{11} & h_{12} & 0 & \cdots & 0 \\ h_{21} & h_{22} & h_{23} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ h_{n-1,1} & h_{n-1,2} & \cdots & h_{n-1,n-1} & h_{n-1,n} \\ h_{n,1} & h_{n,2} & \cdots & h_{n,n-1} & h_{n,n} \end{bmatrix},$$

$$\widetilde{H}=\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ h_{11} & h_{12} & 0 & \ddots & \vdots & 0 \\ h_{21} & h_{22} & h_{23} & \ddots & 0 & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 & 0 \\ h_{n-1,1} & h_{n-1,2} & \cdots & h_{n-1,n-1} & h_{n-1,n} & 0 \\ h_{n,1} & h_{n,2} & \cdots & h_{n,n-1} & h_{n,n} & 1 \end{bmatrix}$$

and

$$\widetilde{H}^{-1}=\left[\begin{array}{c|c} [\alpha]_{n\times 1} & [L]_{n\times n} \\ \hline h & [\beta^T]_{1\times n} \end{array}\right]_{(n+1)\times(n+1)}, \tag{2}$$

then they obtained equalities

$$\det(H)=(-1)^n h.\det(\widetilde{H}), \tag{3}$$

$$L=H^{-1}+h^{-1}\alpha\beta^T \tag{4}$$

and

$$H\alpha+he_n=0 \tag{5}$$

where e_n is n-th column of the identity matrix I_n.

The following lemma can be inferred from (3), (4) and (5).

Lemma 1.1. *Let e_1 be the first column of the identity matrix I_n and matrices L, β, h in (2). Then,*

$$\beta^T H+he_1=0. \tag{6}$$