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p-adic Eisenstein–Kronecker series and non-critical values of p-adic Hecke L-function of an imaginary quadratic field when the conductor is divisible by p

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ABSTRACT

Text. We relate non-critical special values of p-adic L-functions associated to algebraic Hecke characters of an imaginary quadratic number field with class number one to p-adic Eisenstein–Kronecker series constructed as the Coleman function, when the conductors of the algebraic Hecke characters are divisible by p.

Video. For a video summary of this paper, please click here or visit http://youtu.be/AZemqgfp5pQ.

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0. Introduction

Let p be a rational prime number. The purpose of this article is to relate non-critical values of the p-adic L-functions associated to algebraic Hecke characters whose conductors are divisible by p of an imaginary quadratic field with class number 1 to p-adic Eisenstein–Kronecker series. We expect to use this result in the future to consider the p-adic Beilinson conjecture as in [2] for the corresponding Hecke character. The Beilinson conjectures about special values of L-functions are a vast generalization of the class number formula for Dedekind zeta function (see [5]), which state that non-critical values of L-functions of an algebraic variety can be expressed using invariants arising from the Beilinson regulator map. More generally, these conjectures can be formulated for motives. The p-adic Beilinson conjectures are the p-adic analogues of the Beilinson conjectures, which state that non-critical values of p-adic L-functions of an algebraic variety may be concretely expressed by invariants arising from the syntomic regulator. The p-adic Beilinson conjectures were formulated and proved by Gros in the case of Dirichlet motives [17,18] and were generalized to p-adic L-functions of motives by Perrin-Riou (see [25, §4.2]). For p-adic L-functions of Abelian Artin motives, Coleman related special values of these p-adic L-functions to p-adic polylogarithms defined by using his theory of p-adic integration [12]. The polylogarithms have a motivic interpreter defined by Beilinson and Deligne. In other words, the Beilinson conjectures and their p-adic analogues suggest that special values of L-functions and p-adic L-functions may be expressed by using motivic elements.

The p-adic L-functions of algebraic Hecke characters of our case was first constructed by Manin and Višik [24], N. Katz [21], R.I. Yager [31], and de Shalit [15]. Bannai and Kobayashi gave a different construction of these p-adic L-functions using the Kronecker theta functions associated to the Poincaré bundle (see [4, §2]). In addition, Bannai, Kobayashi, and Tsuji expressed the elliptic polylogarithms on an elliptic curve with complex multiplication in terms of the "Eisenstein–Kronecker series" (for details, see [32]), by using the Kronecker theta functions (see [3, Theorem 1.17]). Deuring's theorem indicates that L-functions of an elliptic curve with complex multiplication by the integer ring of an imaginary quadratic field can be expressed as Hecke L-functions of an imaginary quadratic field. Then the p-adic L-functions of an elliptic curve with complex multiplication become the p-adic L-functions of an imaginary quadratic field associated to algebraic Hecke characters.

When the conductors of algebraic Hecke characters are not divisible by p, Bannai, Kobayashi, and Tsuji related non-critical values of p-adic L-functions of an imaginary