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On Chen's theorem (II) ☆

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Abstract

Let N be a sufficiently large even integer and $S(N)$ denote the number of solutions of the equation

$$N = p + P_2,$$

where p denotes a prime and P_2 denotes an almost-prime with at most two prime factors. In this paper we obtain

$$S(N) > \frac{0.867C(N)N}{\log^2 N},$$

where

$$C(N) = \prod_{p>2}\left(1 - \frac{1}{(p-1)^2}\right) \prod_{\substack{p|N\\ p>2}} \frac{p-1}{p-2},$$

and thus improved the previous result

$$S(N) > \frac{0.836C(N)N}{\log^2 N}$$

due to J. Wu.

MSC: 11N05

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1. Introduction

In 1966 Jingrun Chen [4] announced his remarkable theorem—Chen's theorem: let N be a sufficiently large even integer and $S(N)$ denote the number of solutions of the equation

$$N = p + P_2,$$

where and in what follows p, with or without subscript, is a prime and P_2 is an almost-prime with at most two prime factors, then

$$S(N) > \frac{0.67C(N)N}{\log^2 N},$$

where

$$C(N) = \prod_{p>2}\left(1 - \frac{1}{(p-1)^2}\right) \prod_{\substack{p|N\\ p>2}} \frac{p-1}{p-2},$$

and the detail was published in [5]. The original proof of Jingrun Chen was simplified by Pan, Ding and Wang [17], Halberstam and Richert [12], Halberstam [11], Ross [19]. In [12] Halberstam and Richert announced that they obtained the constant 0.689 and a detail proof was given in [11]. In p. 338 of [12] it says: "It would be interesting to know whether the more elaborate weighting procedure could be adapted to the numerical improvements and could be important." In 1978 Jingrun Chen [7] introduced a new sieve procedure to show that

$$S(N) > \frac{0.81C(N)N}{\log^2 N}.$$

In 2000 Y.C. Cai and M.G. Lu [2] refined Jingrun Chen's sieve procedure and obtained

$$S(N) > \frac{0.8285C(N)N}{\log^2 N}.$$

In 2004 J. Wu [25] improved the argument in [2] and obtained

$$S(N) > \frac{0.836C(N)N}{\log^2 N}.$$

In this paper we shall combine the sieve procedures in [2,8,22,25] to prove

Theorem 1.

$$S(N) > \frac{0.867C(N)N}{\log^2 N}.$$