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Palindromic automorphisms of free nilpotent groups

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A B S T R A C T

In this paper, we initiate the study of palindromic automorphisms of groups that are free in some variety. More specifically, we define palindromic automorphisms of free nilpotent groups and show that the set of such automorphisms is a group. We find a generating set for the group of palindromic automorphisms of free nilpotent groups of step 2 and 3. In particular, we obtain a generating set for the group of central palindromic automorphisms of these groups. In the end, we determine central palindromic automorphisms of free nilpotent groups of step 3 which satisfy the necessary condition of Bryant–Gupta–Levin–Mochizuki for a central automorphism to be tame.

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1. Introduction

Let $\mathcal{M}$ be a variety of groups, and F be a group that is free in $\mathcal{M}$. Let $X = \{x_1, \ldots, x_n\}$ be a basis of F. A reduced word $w \doteq x_{i_1} x_{i_2} \ldots x_{i_m}$ in $X^{\pm 1}$ is called a *palindrome* in the alphabet $X^{\pm 1}$ if it is equal as a word to its reverse word $\overline{w} \doteq x_{i_m} \ldots x_{i_2} x_{i_1}$, where by $\doteq$ we denote equality of letter by letter. An element $g \in F$ is called a *palindrome* if it can be represented by some word in the alphabet $X^{\pm 1}$, which is a palindrome. Note that this definition depends on the generating set X of F.

In [6], Collins defined and investigated palindromic automorphisms of absolutely free groups. Following Collins, we say that an automorphism ϕ of F is a *palindromic automorphism* if x_i^{ϕ} is a palindrome with respect to X for each $1 \leq i \leq n$. It is not difficult to check that the product of two palindromic automor-

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phisms is again a palindromic automorphism. We denote the monoid of palindromic automorphisms of F by $\Pi A(F)$.

Let $p_1, \dots, p_n \in F$. An automorphism ϕ of F of the form

$$\phi : x_i \mapsto \overline{p_i} x_i p_i \text{ for each } 1 \leq i \leq n,$$

is called an *elementary palindromic automorphism*. The sub-monoid of elementary palindromic automorphisms of F is denoted by $E\Pi A(F)$.

If $F = F_n$ with basis $\{x_1, \dots, x_n\}$, then for each $1 \leq i \leq n$ and $1 \leq j \leq n-1$, the maps

$$t_i : \begin{cases} x_i \longmapsto x_i^{-1} \\ x_k \longmapsto x_k & \text{for } k \neq i \end{cases}$$

and

$$\alpha_{j,j+1} : \begin{cases} x_j \longmapsto x_{j+1} \\ x_{j+1} \longmapsto x_j \\ x_k \longmapsto x_k & \text{for } k \neq j, \end{cases}$$

are automorphisms of F_n. The group

$$\Omega S_n(F_n) := \langle t_i, \alpha_{j,j+1} \mid \ 1 \leq i \leq n \text{ and } 1 \leq j \leq n-1 \rangle$$

is a subgroup of $Aut(F_n)$ (see, for example, [17]). Now, if the group F is free in some variety and has rank n, then there is a homomorphism $Aut(F_n) \to Aut(F)$. The image of $\Omega S_n(F_n)$ under this homomorphism is called the *extended symmetric group* and denoted by $\Omega S_n(F)$. For simplicity, we denote generators of $\Omega S_n(F)$ by t_i, $\alpha_{j,j+1}$, which act on a basis of F as above. Clearly, $E\Pi A(F)$ and $\Omega S_n(F)$ generate $\Pi A(F)$ as a monoid and $\Pi A(F) = E\Pi A(F) \rtimes \Omega S_n(F)$. Let $IA(F)$ denote the group of those automorphisms of F that induce identity on the abelianization F/F' of F. Let $PI(F) = E\Pi A(F) \cap IA(F)$ denote the sub-monoid of *palindromic IA-automorphisms* of F.

If $F = F_n$ is a free group, then Collins [6] obtained a generating set for $\Pi A(F_n)$. In particular, Collins proved that $E\Pi A(F_n)$ is a group generated by μ_{ij} for $1 \leq i \neq j \leq n$, where

$$\mu_{ij} : \begin{cases} x_i \longmapsto x_j x_i x_j \\ x_k \longmapsto x_k & \text{for } k \neq i. \end{cases}$$

In the same paper [6], Collins conjectured that $E\Pi A(F_n)$ is torsion free for each $n \geq 2$. Using geometric techniques, Glover and Jensen [11] proved this conjecture and also calculated the virtual cohomological dimension of $\Pi A(F_n)$. Extending this work in [13], Jensen, McCommand and Meier computed the Euler characteristic of $\Pi A(F_n)$ and $E\Pi A(F_n)$. In [16], Piggott and Ruane constructed Markov languages of normal forms for $\Pi A(F_n)$ using methods from logic theory. In [18,19], Nekritsukhin investigated some basic group theoretic questions about $\Pi A(F_n)$. In particular, he studied involutions and center of $\Pi A(F_n)$. In a recent paper [10], Fullarton obtained a generating set for the palindromic IA-automorphism group $PI(F_n)$. This was obtained by constructing an action of $PI(F_n)$ on a simplicial complex modeled on the complex of partial bases due to Day and Putman [7]. The papers [11] and [10] indicate a deep connection between palindromic automorphisms of free groups and geometry. Recently, Bardakov, Gongopadhyay and Singh [3] investigated many algebraic properties of $\Pi A(F_n)$. In particular, they obtained conjugacy classes of involutions in $\Pi A(F_2)$ and investigated residual nilpotency of $\Pi A(F_n)$. They also refined a result of Fullarton [10] by proving that $PI(F_n) = IA(F_n) \cap E\Pi A'(F_n)$.

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