Journal of Pure and Applied Algebra 219 (2015) 5511–5538

Contents lists available at ScienceDirect

Journal of Pure and Applied Algebra

www.elsevier.com/locate/jpaa

Partial actions of weak Hopf algebras: Smash product, globalization and Morita theory

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A R T I C L E I N F O

Article history:
Received 31 August 2014
Received in revised form 14 April 2015
Available online 2 June 2015
Communicated by C. Kassel

MSC:
Primary: 16T99; secondary: 20L05

A B S T R A C T

In this paper we introduce the notion of partial action of a weak Hopf algebra on algebras, unifying the notions of partial group action [11], partial Hopf action [2,3,9] and partial groupoid action [4]. We construct the fundamental tools to develop this new subject, namely, the partial smash product and the globalization of a partial action, as well as we establish a connection between partial and global smash products via the construction of a surjective Morita context. In particular, in the case that the globalization is unital, these smash products are Morita equivalent. We show that there is a bijective correspondence between globalizable partial groupoid actions and symmetric partial groupoid algebra actions, extending similar result for group actions [9]. Moreover, as an application we give a complete description of all partial actions of a weak Hopf algebra on its ground field, which suggests a method to construct more general examples.

1. Introduction

Partial actions of groups on algebras were introduced in the literature by R. Exel in [12]. His main purpose in that paper was to develop a method that allowed to describe the structure of C^*-algebras under actions of the circle group. The first approach of partial group actions on algebras, in a purely algebraic context, appears later in a paper by M. Dokuchaev and R. Exel [11].

Partial group actions can be easily obtained by restriction from the global ones, and this fact stimulated the interest in knowing under what conditions (if any) a given partial group action is of this type. In the topological context this question was dealt with by F. Abadie in [1]. The algebraic version of a globalization (or enveloping action) of a partial group action, as well as the study about its existence, was also considered by M. Dokuchaev and R. Exel in [11]. A nice approach on the relevance of the relationship between partial and global group actions, in several branches of mathematics, can be seen in [10].

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[1] The first and the third authors were partially supported by CNPq, Brazil.

http://dx.doi.org/10.1016/j.jpaa.2015.05.031
0022-4049/

As a natural task, S. Caenepeel and K. Janssen [9] extended the notion of partial group action to the setting of Hopf algebras and developed a theory of partial (co)actions of Hopf algebras, as well as a partial Hopf–Galois theory. Based on Caenepeel–Janssen's work, E. Batista and M. Alves in [2,3] showed that every partial action of a Hopf algebra has a globalization and that the corresponding partial and global smash products are related by a surjective Morita context. At almost the same time D. Bagio and A. Paques developed a theory of partial groupoid actions extending, in particular, results of M. Dokuchaev and R. Exel in [11] about partial group actions and their globalizations.

Actions of weak Hopf algebras is the precise context to unify all these mentioned above theories, and this is our main purpose.

In this paper we deal with actions of weak Hopf algebras and extend to this setting many of the results above mentioned. As it is well known, Hopf algebras and groupoid algebras are perhaps the simplest examples of weak Hopf algebras. The weak Hopf algebra theory has been started at the end of the 90s by G. Böhm, F. Nill and K. Szlachányi [5–7,17,19].

One of our main goals is to show that the notion of globalization can be extended to partial module algebras over weak Hopf algebras. We succeed to prove that every (left) partial module algebra over weak Hopf algebras has a globalization (also called enveloping action), extending the corresponding results on partial Hopf actions obtained by E. Batista and M. Alves in [2] and [3]. We also prove the existence of minimal globalizations and that any two of them are equivalent, as well as that any globalization is a homomorphic preimage of a minimal one (see Section 5).

The other one is to ensure that the partial version of the smash product, as introduced by D. Nikshych in [16], can also be obtained (see Section 6). The hardest task here is to show that such a partial smash product is well defined, just because the tensor product, in this case, is not over the ground field. The usual theory does not fit into our context since the definitions for partial structures are a bit different. The existence of partial smash products allows us to construct a surjective Morita context relating them with the corresponding global ones (see Section 7).

As an application, we describe completely all the partial actions of a weak Hopf algebra on its ground field, which also suggests the construction of other examples of such partial actions, different from the canonical ones (see Section 4). We also analyze the relation between partial groupoid actions, as introduced in [4], and partial actions of groupoid algebras, showing how partial group actions, in particular, and partial groupoid actions, in general, fit into this new context (see Section 3).

In Section 2 we present the definitions and basic results that we will need in the sequel.

Throughout, $\Bbbk$ will denote a field and every $\Bbbk$-algebra is assumed to be associative and unital, unless otherwise stated. Unadorned $\otimes$ means $\otimes_{\Bbbk}$.

2. Partial actions of weak Hopf algebras

2.1. Weak Hopf algebras

We start recalling the definition and some of the properties of a weak Hopf algebra over a field $\Bbbk$. For more about it we refer to [6].

Definition 2.1. A sixtuple $(H, m, u, \Delta, \varepsilon, S)$ is a weak Hopf algebra, with antipode S, if:

(i) $(H,\ m,\ u)$ is a $\Bbbk$-algebra,
(ii) $(H,\ \Delta,\ \varepsilon)$ is a $\Bbbk$-coalgebra,
(iii) $\Delta(kh) = \Delta(k)\Delta(h)$, for all $h, k \in H$,
(iv) $\varepsilon(kh_1)\varepsilon(h_2g) = \varepsilon(khg) = \varepsilon(kh_2)\varepsilon(h_1g)$,
(v) $(1_H\otimes\Delta(1_H))(\Delta(1_H)\otimes 1_H) = \Delta^2(1_H) = (\Delta(1_H)\otimes 1_H)(1_H\otimes\Delta(1_H))$,