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Quantum automorphism groups and $SO(3)$-deformations

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ABSTRACT

We show that any compact quantum group having the same fusion rules as the ones of $SO(3)$ is the quantum automorphism group of a pair (A, φ), where A is a finite dimensional C^*-algebra endowed with a homogeneous faithful state. We also study the representation category of the quantum automorphism group of (A, φ) when φ is not necessarily positive, generalizing some known results, and we discuss the possibility of classifying the cosemisimple (not necessarily compact) Hopf algebras whose corepresentation semi-ring is isomorphic to that of $SO(3)$.
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1. Introduction and main results

The quantum automorphism group of a measured finite dimensional C^*-algebra (A, φ) (i.e. a finite-dimensional C^*-algebra A endowed with a faithful state φ) has been defined by Wang in [24] as the universal object in the category of compact quantum groups acting on (A, φ). The corresponding compact Hopf algebra is denoted by $A_{\mathbf{aut}}(A, \varphi)$.

The structure of $A_{\mathbf{aut}}(A, \varphi)$ depends on the choice of the measure φ, and the representation theory of this quantum group is now well understood [2,3], provided a good choice of φ has been done, namely that φ is a δ-form (we shall say here that φ is homogeneous, and that (A, φ) is a homogeneous measured C^*-algebra). Banica's main result in [2,3] is that if φ is homogeneous and $\dim(A) \geqslant 4$, then $A_{\mathbf{aut}}(A, \varphi)$ has the same corepresentation semi-ring as $SO(3)$. See also [12]. The result can be further extended to show that the corepresentation category of $A_{\mathbf{aut}}(A, \varphi)$ is monoidally equivalent to the representation category of a quantum $SO(3)$-group at a well chosen parameter, see [13].

Then a natural question, going back to [2,3] and formally asked in [4], is whether any compact quantum group with the same fusion rules as $SO(3)$ is the quantum automorphism group of an appropriate measured finite-dimensional C^*-algebra. The main result in this paper is a positive answer to this question.

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Theorem 1.1. *Let H be a compact Hopf algebra with corepresentation semi-ring isomorphic to that of $SO(3)$. Then there exists a finite dimensional homogeneous measured C^*-algebra (A, φ) with $\dim(A) \geqslant 4$ such that $H \simeq A_{\mathbf{aut}}(A, \varphi)$.*

Recall that if G is a reductive algebraic group, a G-deformation is a cosemisimple Hopf algebra H such that $\mathcal{R}^+(H) \simeq \mathcal{R}^+(\mathcal{O}(G))$, where $\mathcal{R}^+$ denotes the corepresentation semi-ring. The problem of the classification of G-deformations has been already studied for several algebraic groups: see [25,1,20,7] for $SL(2)$, [19,17] for $GL(2)$, and [18] for $SL(3)$. Thus Theorem 1.1 provides the full description of the compact $SO(3)$-deformations.

The next natural step is then to study the non-compact $SO(3)$-deformations. For this purpose we study the comodule category of $A_{\mathbf{aut}}(A, \varphi)$ with φ non-necessarily positive and give a generalization of the results from [2,3,8,13] (together with independent proof of these results), as follows (see Section 2 for the relevant definitions).

Theorem 1.2. *Let (A, φ) be a finite dimensional, semisimple algebra endowed with a normalizable measure φ, with $\dim A \geqslant 4$. Then there exists a $\mathbb{C}$-linear equivalence of monoidal categories*

$$\mathrm{Comod}\big(A_{\mathbf{aut}}(A, \varphi)\big) \simeq^{\otimes} \mathrm{Comod}\big(\mathcal{O}\big(SO_q(3)\big)\big)$$

between the comodule categories of $A_{\mathbf{aut}}(A, \varphi)$ and $\mathcal{O}(SO_q(3))$ respectively, for some well-chosen $q \in \mathbb{C}^*$.

We have not been able to show that all $SO(3)$-deformations arise as quantum automorphism groups as in the previous theorem. However see Section 5 for partial results in this direction. Note that the monoidal reconstruction theorem of Tuba–Wenzl [23], which discuss the related but non-equivalent problem of determining the braided semisimple tensor categories of type B, cannot be used in our setting, where the existence of a braiding is not assumed.

This paper is organized as follows: in Section 2, we fix some notations and definitions, state some basic facts about compact Hopf algebras, finite dimensional algebras and we recall the construction of the quantum automorphism group of a finite dimensional, semisimple, measured algebra. Theorem 1.1 is proved in Section 3, thanks to a careful study of the fusion rules of $SO(3)$. In Section 4, we prove Theorem 1.2 by building a cogroupoid linking these Hopf algebras and studying its connectedness and in Section 5, we prove some classification results about Hopf algebras having a corepresentation semi-ring isomorphic to that of $SO(3)$.

2. Preliminaries

2.1. Compact Hopf algebras

Let us recall the definition of a compact Hopf algebra (see [15]):

Definition 2.1.

1. A Hopf $*$-algebra is a Hopf algebra H which is also a $*$-algebra and such that the comultiplication is a $*$-homomorphism.
2. If $x = (x_{ij})_{1 \le i,j \le n} \in M_n(H)$ is a matrix with coefficient in H, the matrix $(x^*_{ij})_{1 \le i,j \le n}$ is denoted by $\bar{x}$, while $\bar{x}^t$, the transpose matrix of $\bar{x}$, is denoted by x^*. The matrix x is said to be unitary if $x^* x = I_n = x x^*$.

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