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Semi-invariants for concealed-canonical algebras

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ABSTRACT

In this paper we generalize known descriptions of rings of semi-invariants for regular modules over Euclidean and canonical algebras to arbitrary concealed-canonical algebras.

Throughout the paper $\Bbbk$ is a fixed algebraically closed field. By $\mathbb{Z}$, $\mathbb{N}$ and $\mathbb{N}_+$ we denote the sets of the integers, the non-negative integers and the positive integers, respectively. Finally, if $i, j \in \mathbb{Z}$, then $[i, j] := \{k \in \mathbb{Z} \mid i \leqslant k \leqslant j\}$ (in particular, $[i, j] = \varnothing$ if $i > j$).

0. Introduction

Concealed-canonical algebras have been introduced by Lenzing and Meltzer [22] as a generalization of Ringel's canonical algebras [26]. An algebra is called concealed-canonical if it is isomorphic to the endomorphism ring of a tilting bundle over a weighted projective line. The concealed-canonical algebras can be characterized as the algebras which possess sincere separating exact subcategory [23] (see also [28]). Together with tilted algebras [7,20], the concealed-canonical algebras form two most prominent classes of quasi-tilted algebras [19]. Moreover, according to a famous result of Happel [18], every quasi-tilted algebra is derived equivalent either to a tilted algebra or to a concealed-canonical algebra.

Despite investigations of a structure of the categories of modules over concealed-canonical algebras, geometric problems have been studied for this class of algebras (see for example [2,3,6,14,15,17,29]). Often these problems were studied for canonical algebras only and sometimes the authors restrict their attention to the concealed-canonical algebras of tame representation type.

In the paper we study a problem, which has been already investigated in the case of canonical algebras. Namely, given a concealed-canonical algebra Λ and a module R, which is a direct sum of modules from a sincere separating exact subcategory of $\operatorname{mod}\Lambda$, we want to describe a structure of the ring of semi-invariants

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associated with Λ and the dimension vector of R. This problem has been solved provided Λ is a canonical algebra and R comes from a distinguished sincere separating exact subcategory of $\operatorname{mod} \Lambda$ (the answers have been obtained independently by Skowroński and Weyman [29] and Domokos and Lenzing [14,15]). This problem has also been solved for another class of concealed-canonical algebras, namely the path algebras of Euclidean quivers [30] (see also [12,27]). Some results in the case of tubular algebras were also obtained [31].

The obtained results are very similar, although the methods used in the proof are completely different. The aim of my paper is to obtain a unified proof of the above results, which would generalize to an arbitrary concealed-canonical algebra. This aim is achieved if the characteristic of $\Bbbk$ equals 0. If $\operatorname{char} \Bbbk > 0$, then we show that an analogous result is true if we study the semi-invariants which are the restrictions of the semi-invariants on the ambient affine space. The precise formulation of the obtained results can be found in Section 6. In particular we prove that the studied rings of semi-invariants are always complete intersections, and are polynomial rings if the considered dimension vector is "sufficiently big".

The paper is organized as follows. In Section 1 we introduce a setup of quivers and their representations, which due to a result of Gabriel [16] is an equivalent way of thinking about algebras and modules. Next, in Section 2 we gather facts about concealed-canonical algebras (equivalently, quivers). In Section 3 we introduce semi-invariants and present their basic properties. Next, in Section 4 we study the semi-invariants in the case of concealed-canonical quivers more closely. Section 5 is devoted to presentation of necessary facts about the Kronecker quiver, which is the minimal concealed-canonical quiver. Finally, in Section 6 we present and proof the main result.

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1. Quivers and their representations

By a quiver Δ we mean a finite set Δ_0 (called the set of vertices of Δ) together with a finite set Δ_1 (called the set of arrows of Δ) and two maps $s, t : \Delta_1 \to \Delta_0$, which assign to each arrow α its starting vertex $s\alpha$ and its terminating vertex $t\alpha$, respectively. By a path of length $n \in \mathbb{N}_+$ in a quiver Δ we mean a sequence $\sigma = (\alpha_1, \ldots, \alpha_n)$ of arrows such that $s\alpha_i = t\alpha_{i+1}$ for each $i \in [1, n-1]$. In the above situation we put $\ell\sigma := n$, $s\sigma := s\alpha_n$ and $t\sigma := t\alpha_1$. We treat every arrow in Δ as a path of length 1. Moreover, for each vertex x we have a trivial path $\mathbf{1}_x$ at x such that $\ell\mathbf{1}_x := 0$ and $s\mathbf{1}_x := x =: t\mathbf{1}_x$. For the rest of the paper we assume that the considered quivers do not have oriented cycles, where by an oriented cycle we mean a path σ of positive length such that $s\sigma = t\sigma$.

Let Δ be a quiver. We define its path category $\Bbbk\Delta$ to be the category whose objects are the vertices of Δ and, for $x, y \in \Delta_0$, the morphisms from x to y are the formal $\Bbbk$-linear combinations of paths starting at x and terminating at y. If ω is a morphism from x to y, then we write $s\omega := x$ and $t\omega := y$. By a representation of Δ we mean a functor from $\Bbbk\Delta$ to the category $\operatorname{mod} \Bbbk$ of finite dimensional vector spaces. We denote the category of representations of Δ by $\operatorname{rep} \Delta$. Observe that every representation of Δ is uniquely determined by its values on the vertices and the arrows. Given a representation M of Δ we denote by $\mathbf{dim}\, M$ its dimension vector defined by the formula $(\mathbf{dim}\, M)(x) := \dim_\Bbbk M(x)$, for $x \in \Delta_0$. Observe that $\mathbf{dim}\, M \in \mathbb{N}^{\Delta_0}$ for each representation M of Δ. We call the elements of $\mathbb{N}^{\Delta_0}$ dimension vectors. A dimension vector $\mathbf{d}$ is called sincere if $\mathbf{d}(x) \neq 0$ for each $x \in \Delta_0$.

By a relation in a quiver Δ we mean a $\Bbbk$-linear combination of paths of lengths at least 2 having a common starting vertex and a common terminating vertex. Note that each relation in a quiver Δ is a morphism in $\Bbbk\Delta$. A set $\mathfrak{R}$ of relations in a quiver Δ is called minimal if $\langle \mathfrak{R} \setminus \{\rho\} \rangle \neq \langle \mathfrak{R} \rangle$ for each $\rho \in \mathfrak{R}$, where for a set $\mathfrak{X}$ of morphisms in Δ we denote by $\langle \mathfrak{X} \rangle$ the ideal in $\Bbbk\Delta$ generated by $\mathfrak{X}$. Observe that each minimal set of relations is finite. By a bound quiver $\boldsymbol{\Delta}$ we mean a quiver Δ together with a minimal set $\mathfrak{R}$ of relations. Given a bound quiver $\boldsymbol{\Delta}$ we denote by $\Bbbk\boldsymbol{\Delta}$ its path category, i.e. $\Bbbk\boldsymbol{\Delta} := \Bbbk\Delta / \langle \mathfrak{R} \rangle$. By a representation