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Neighborhood radius estimation for Arnold's miniversal deformations of complex and p-adic matrices

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ABSTRACT

V.I. Arnold (1971) constructed a simple normal form to which all complex matrices B in a neighborhood U of a given square matrix A can be reduced by similarity transformations that smoothly depend on the entries of B. We calculate the radius of the neighborhood U. A.A. Mailybaev (1999, 2001) constructed a reducing similarity transformation in the form of Taylor series; we construct this transformation by another method. We extend Arnold's normal form to matrices over the field $\mathbb{Q}_p$ of p-adic numbers and the field $\mathbb{F}((T))$ of Laurent series over a field $\mathbb{F}$.

1. Introduction

The reduction of a complex matrix to its Jordan form is an unstable operation: both the Jordan form and a reduction transformation depend discontinuously on the entries of

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the original matrix. Arnold [1] (see also [2,3]) constructed a *miniversal deformation* of a square complex matrix A; i.e., a simple normal form $B_{\rm arn}$ to which all complex matrices B close to A can be reduced by similarity transformations that smoothly depend on the entries of B.

More precisely: Arnold supposes without restriction that A is a Jordan canonical matrix and reduces all matrices B in a neighborhood U of A to the form $B_{\rm arn}$ by a smooth similarity transformation that acts identically on A. Klimenko and Sergeichuk [14] described this reduction in detail.

Many applications of Arnold's normal form in different areas of mathematics are given in more than 120 articles from the Mathematical Reviews Citation Database that cite [1]. We mention only Mailybaev's articles [16–18] in which applications of miniversal deformations are based on the fact that the spectrum of $B \in U$ and $B_{\rm arn}$ coincide but $B_{\rm arn}$ has a simple form. Mailybaev also constructed a smooth similarity transformation (in the form of Taylor series) that transforms all $B \in U$ to $B_{\rm arn}$.

Galin [9] (see also [3, § 30E]) obtained miniversal deformations of real matrices by realification of Arnold's miniversal deformations of complex matrices. Simpler miniversal deformations of real matrices were given by Garcia-Planas and Sergeichuk [11].

The main results of our paper are formulated in Theorem 7:

- *We extend Arnold's normal form of complex matrices to matrices over any field that is complete with respect to a nontrivial absolute value* (in particular, over *the field $\mathbb{Q}_p$ of p-adic numbers* and *the field $\mathbb{F}((T))$ of Laurent series over a field $\mathbb{F}$*; see Example 6). We use the Frobenius canonical form for similarity over an arbitrary field instead of the Jordan canonical form.
- *Over such a field, we construct a smooth similarity transformation that transforms all $B \in U$ to $B_{\rm arn}$*. Our method differs from the method developed by Mailybaev [16–18].
- *We give the neighborhood U in an explicit form*, which is important for applications. As far as we know, the estimate of the radius of U was unknown even for complex matrices.

Miniversal deformations, reducing transformations, and neighborhood radius estimations for complex matrices under congruence and *congruence were given by Dmytryshyn, Futorny, and Sergeichuk [6,7].

2. Preliminaries

In this section, we recall some definitions and known facts.

2.1. Arnold's miniversal deformations of Jordan matrices

The similarity class of an $n \times n$ complex A in a small neighborhood of A can be obtained by a very small deformation of the affine matrix space