

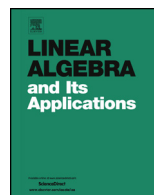


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# Edge perturbation on graphs with clusters: Adjacency, Laplacian and signless Laplacian eigenvalues

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## ABSTRACT

Let  $G$  be a simple undirected graph of order  $n$ . A cluster in  $G$  of order  $c$  and degree  $s$ , is a pair of vertex subsets  $(C, S)$ , where  $C$  is a set of cardinality  $|C| = c \geq 2$  of pairwise co-neighbor vertices sharing the same set  $S$  of  $s$  neighbors. Assuming that the graph  $G$  has  $k \geq 1$  clusters  $(C_1, S_1), \dots, (C_k, S_k)$ , consider a family of  $k$  graphs  $H_1, \dots, H_k$  and the graph  $G(H_1, \dots, H_k)$  which is obtained from  $G$  after adding the edges of the graphs  $H_1, \dots, H_k$  whose vertex set of each  $H_j$  is identified with  $C_j$ , for  $j = 1, \dots, k$ . The Laplacian eigenvalues of  $G(H_1, \dots, H_k)$  remain the same, independently of the graphs  $H_1, \dots, H_k$ , with the exception of  $|C_1| + \dots + |C_k| - k$  of them. These new Laplacian eigenvalues are determined using a unified approach which can also be applied to the determination of a same number of adjacency and signless Laplacian eigenvalues when the graphs  $H_1, \dots, H_k$  are regular. The Faria's lower bound on the multiplicity of the Laplacian eigenvalue 1 of a graph with pendant vertices is generalized. Furthermore, the algebraic connectivity and the Laplacian index of  $G(H_1, \dots, H_k)$  remain the same, independently of the graphs  $H_1, \dots, H_k$ .

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## 1. Introduction

The Laplacian and signless Laplacian spectra of a graph  $G$  with a cluster  $(C, S)$ , where  $C$  is a set of cardinality  $|C| \geq 2$  of pairwise co-neighbor vertices sharing the same set  $S$  of vertices, were studied in [2], as well as when those graphs are perturbed by adding edges between pairs of co-neighbor vertices. The particular case of graphs with pendant vertices perturbed by adding edges linking them, was considered in [4,9–11]. The effects on several spectral invariants were analyzed in [9] and [10]. In this paper, we consider the adjacency, Laplacian and signless Laplacian matrix of a graph in a unified way, when a graph  $G$  of order  $n$  has  $k \geq 1$  clusters  $(C_1, S_1), \dots, (C_k, S_k)$ . With this approach, several spectral results are deduced. Namely, identifying the vertex sets of a family of graphs  $H_1, \dots, H_k$  with the vertices of  $G$  such that  $V(H_1) = C_1, \dots, V(H_k) = C_k$  and denoting the obtained graph by  $G(H_1, \dots, H_k)$  we may conclude that the Laplacian index and the algebraic connectivity of both graphs,  $G$  and  $G(H_1, \dots, H_k)$ , are the same. On the other hand, when the family of  $k$  graphs  $H_1, \dots, H_k$  varies, all the Laplacian eigenvalues remain the same with the exception of  $|C_1| + \dots + |C_k| - k$  of them. These new Laplacian eigenvalues are determined using a unified approach which can also be applied to the determination of  $|C_1| + \dots + |C_k| - k$  adjacency and signless Laplacian eigenvalues when the graphs  $H_1, \dots, H_k$  are regular. Furthermore, we generalize the Faria's lower bound on the multiplicity of the Laplacian eigenvalue 1 of graphs with pendant vertices and this generalization is straightforward extended to the signless Laplacian eigenvalue 1 and to the adjacency eigenvalue 0. It will be also proved that the algebraic connectivity and Laplacian index of  $G(H_1, \dots, H_k)$  remain the same, independently of the graphs  $H_1, \dots, H_k$ .

The paper is organized as follows. The next section is devoted to introduce the notation and to recall the basic concepts. A general result on the spectral effects in graphs with a cluster by adding edges among pairwise co-neighbor vertices is obtained in Section 3. This result is generalized to graphs with  $k \geq 1$  pairwise disjoint clusters in Section 4. In Section 5 a generalization of Faria's lower bound on the multiplicity of 1 as Laplacian eigenvalue is deduced and it is also proved that the Laplacian index and the algebraic connectivity remains the same when the family of graphs  $H_1, \dots, H_k$  varies in  $G(H_1, \dots, H_k)$ , assuming that  $G$  has  $k$  clusters.

## 2. Notation and basic definitions

Let  $G = (V(G), E(G))$  be a simple undirected graph on  $n$  vertices with vertex set  $V(G)$  and edge set  $E(G)$ . Let  $D(G)$  be the diagonal matrix of order  $n$  whose  $(i, i)$ -entry is the degree  $d_i$  of the  $i$ -th vertex of  $G$  and let  $A(G)$  be the adjacency matrix of  $G$ . The matrices  $L(G) = D(G) - A(G)$  and  $L^+(G) = D(G) + A(G)$  are the Laplacian and signless Laplacian matrix of  $G$ , respectively.  $L(G)$  and  $L^+(G)$  are both positive semidefinite and  $(0, \mathbf{1}_n)$  is an eigenpair of  $L(G)$  where  $\mathbf{1}_n$  is the all 1- vector of size  $n$ . Fiedler [6] proved that  $G$  is a connected graph if and only if the second smallest eigenvalue of  $L(G)$  is

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